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A Simple Construction of the Golden Section

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Abstract. We construct the golden section by drawing 5 circular arcs.

We denote by $P(Q)$ the circle with P as center and PQ as radius. Figure 1 shows two circles $A(B)$ and $B(A)$ intersecting at C and D. The line AB intersects the circles again at E and F. The circles $A(F)$ and $B(E)$ intersect at two points X and Y. It is clear that C, D, X, Y are on a line. It is much more interesting to note that D divides the segment CX in the golden ratio, *i.e.*,

$$\frac{CD}{CX} = \frac{\sqrt{5}-1}{2}.$$

This is easy to verify. If we assume AB of length 2, then $CD = 2\sqrt{3}$ and $CX = \sqrt{15}+\sqrt{3}$. From these,

$$\frac{CD}{CX} = \frac{2\sqrt{3}}{\sqrt{15}+\sqrt{3}} = \frac{2}{\sqrt{5}+1} = \frac{\sqrt{5}-1}{2}.$$

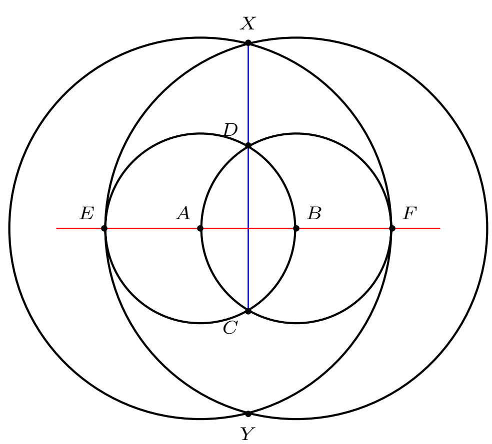

Figure 1

This shows that to construct three collinear points in golden section, we need four circles and one line. It is possible, however, to replace the line AB by a circle, say $C(D)$. See Figure 2. Thus, *the golden section can be constructed with compass only, in 5 steps*.

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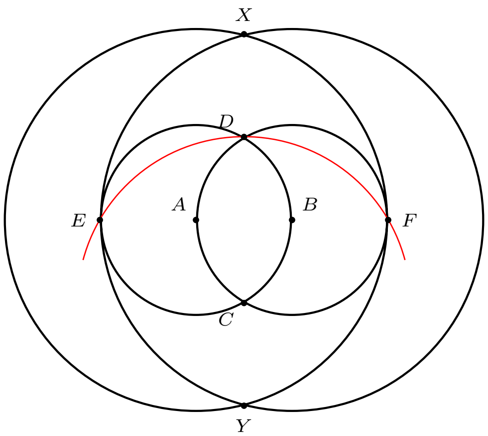

Figure 2

It is interesting to compare this with Figure 3 which also displays the golden section. See [1, p.105, note on 3.5(b)] and [2].[1] Here, ABC is an equilateral triangle. The line joining the midpoints D, E of two sides intersects the circumcircle at F. Then E divides DF in the golden section, *i.e.*,

$$\frac{DE}{DF} = \frac{\sqrt{5}-1}{2}.$$

However, it is unlikely that this diagram can be constructed in fewer than 5 steps, using ruler and compass, or compass alone.

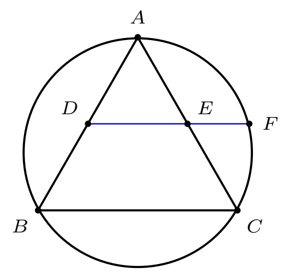

Figure 3

References

[1] D. H. Fowler, *The Mathematics of Plato's Academy*, Oxford University Press, 1988.

[2] G. Odom and J. van de Craats, Elementary Problem 3007, *American Math. Monthly*, 90 (1983) 482; solution, 93 (1986) 572.

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[1] I am indebted to a referee for these references.