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Loci Related to Variable Flanks

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Abstract. Let BR_1R_2C, CR_3R_4A, AR_5R_6B be rectangles built on the sides of a triangle ABC such that the oriented distances $|BR_1|$, $|CR_3|$, $|AR_5|$ are $\lambda\,|BC|$, $\lambda\,|CA|$, $\lambda\,|AB|$ for some real number λ. We explore relationships among the central points of triangles ABC, AR_4R_5, BR_6R_1, and CR_2R_3. Our results extend recent results by Hoehn, van Lamoen, C. Pranesachar and Venkatachala who considered the case when $\lambda = 1$ (with squares erected on sides).

1. Introduction

In recent papers (see [2], [5], and [6]), L. Hoehn, F. van Lamoen, and C. R. Pranesachar and B. J. Venkatachala have considered the classical geometric configuration with squares BS_1S_2C, CS_3S_4A, and AS_5S_6B erected on the sides of a triangle ABC and studied relationships among the central points (see [3]) of the base triangle $\tau = ABC$ and of three interesting triangles $\tau_A = AS_4S_5$, $\tau_B = BS_6S_1$, $\tau_C = CS_2S_3$ (called *flanks* in [5] and *extriangles* in [2]). In order to describe their main results, recall that triangles ABC and XYZ are *homologic* provided that the lines AX, BY, and CZ are concurrent. The point P in which they concur is their *homology center* and the line ℓ containing the intersections of the pairs of lines $(BC,\, YZ)$, $(CA,\, ZX)$, and $(AB,\, XY)$ is their *homology axis*. In this situation we use the notation $ABC \overset{P}{\underset{\ell}{\bowtie}} XYZ$, where ℓ or both ℓ and P may be omitted. Let $X_i = \underline{X}_i(\tau)$, $X_i^j = \underline{X}_i(\tau_j)$ (for $j = A,\, B,\, C$), and $\sigma_i = X_i^A X_i^B X_i^C$, where $\underline{X}_i$ (for $i = 1, \ldots$) is any of the triangle central point functions from Kimberling's lists [3] or [4].

Instead of homologic, homology center, and homology axis many authors use the terms *perspective*, *perspector*, and *perspectrix*. Also, it is customary to use letters I, G, O, H, F, K, and L instead of X_1, X_2, X_3, X_4, X_5, X_6, and X_{20} to denote the incenter, the centroid, the circumcenter, the orthocenter, the center of the nine-point circle, the symmedian (or Grebe-Lemoine) point, and the de Longchamps point (the reflection of H about O), respectively.

In [2] Hoehn proved $\tau \bowtie \sigma_3$ and $\tau \overset{X_j}{\bowtie} \sigma_i$ for $(i,\, j) = (1,\, 1),\, (2,\, 4),\, (4,\, 2)$. In [6] C. R. Pranesachar and B. J. Venkatachala added some new results because they showed that $\tau \overset{X_j}{\bowtie} \sigma_i$ for $(i,\, j) = (1,\, 1),\, (2,\, 4),\, (4,\, 2),\, (3,\, 6),\, (6,\, 3)$. Moreover,

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they observed that if $\tau \overset{X}{\bowtie} X_A X_B X_C$, and Y, Y_A, Y_B, and Y_C are the isogonal conjugates of points X, X_A, X_B, and X_C with respect to triangles τ, τ_A, τ_B, and τ_C respectively, then $\tau \overset{Y}{\bowtie} Y_A Y_B Y_C$. Finally, they also answered negatively the question by Prakash Mulabagal of Pune if $\tau \bowtie XYZ$, where X, Y, and Z are the points of contact of the incircles of triangles τ_A, τ_B, and τ_C with the sides opposite to A, B, and C, respectively.

In [5] van Lamoen said that X_i *befriends* X_j when $\tau \overset{X_j}{\bowtie} \sigma_i$ and showed first that $\tau \overset{X_j}{\bowtie} \sigma_i$ implies $\tau \overset{X_n}{\bowtie} \sigma_m$ where X_m and X_n are the isogonal conjugates of X_i and X_j. Also, he proved that $\tau \overset{X_j}{\bowtie} \sigma_i$ is equivalent to $\tau \overset{X_i}{\bowtie} \sigma_j$, and that $\tau \overset{X_j}{\bowtie} \sigma_i$ for $(i, j) = (1, 1), (2, 4), (3, 6), (4, 2), (6, 3)$. Then he noted that $\tau \overset{K(\frac{\pi}{2}-\phi)}{\bowtie} K(\phi)$, where $K(\phi)$ denotes the homology center of τ and the Kiepert triangle formed by apexes of similar isosceles triangles with the base angle ϕ erected on the sides of ABC. This result implies that $\tau \overset{X_i}{\bowtie} \sigma_i$ for $i = 485, 486$ (Vecten points – for $\phi = \pm\frac{\pi}{4}$), and $\tau \overset{X_j}{\bowtie} \sigma_i$ for $(i, j) = (13, 17), (14, 18)$ (isogonic or Fermat points X_{13} and X_{14} – for $\phi = \pm\frac{\pi}{3}$, and Napoleon points X_{17} and X_{18} – for $\phi = \pm\frac{\pi}{6}$). Finally, van Lamoen observed that the Kiepert hyperbola (the locus of $K(\phi)$) befriends itself; so does its isogonal transform, the Brocard axis OK.

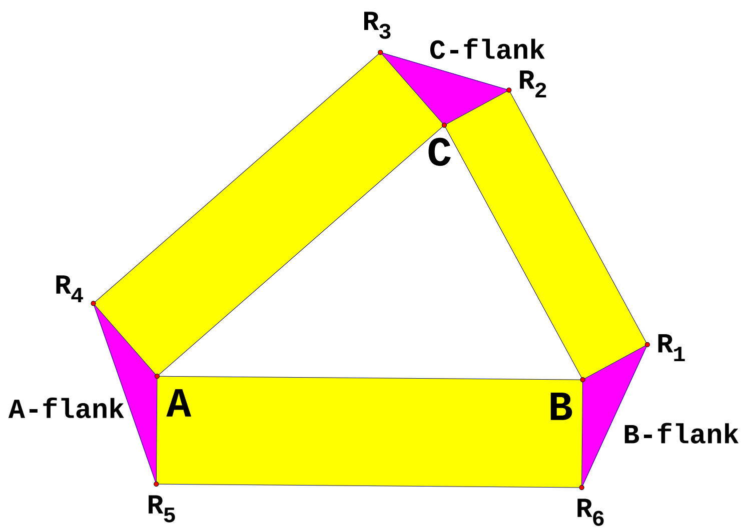

Figure 1. Triangle ABC with three rectangles and three flanks.

The purpose of this paper is to extend and improve the above results by replacing squares with rectangles whose ratio of nonparallel sides is constant. More precisely, let BR_1R_2C, CR_3R_4A, AR_5R_6B be rectangles built on the sides of a triangle ABC such that the oriented distances $|BR_1|$, $|CR_3|$, $|AR_5|$ are $\lambda\,|BC|$,

$\lambda\,|CA|$, $\lambda\,|AB|$ for some real number λ. Let $\tau_A^\lambda = AR_4R_5$, $\tau_B^\lambda = BR_6R_1$, and $\tau_C^\lambda = CR_2R_3$ and let $X_i^j(\lambda)$ and σ_i^λ, for $j = A,\, B,\, C$, have obvious meaning. The most important central points have their traditional notations so that we shall often use these because they might be easier to follow. For example, $H^A(\lambda)$ is the orthocenter of the flank τ_A^λ and σ_G^λ is the triangle $G^A(\lambda)G^B(\lambda)G^C(\lambda)$ of the centroids of flanks.

Since triangles AS_4S_5 and AR_4R_5 are homothetic and the vertex A is the center of this homothety (and similarly for pairs BS_6S_1, BR_6R_1 and CS_2S_3, CR_2R_3), we conclude that $\{A,\, X_i^A,\, X_i^A(\lambda)\}$, $\{B,\, X_i^B,\, X_i^B(\lambda)\}$, and $\{C,\, X_i^C,\, X_i^C(\lambda)\}$ are sets of collinear points so that all statements from [2], [6], and [5] concerning triangles σ_i are also true for triangles σ_i^λ.

However, since in our approach instead of a single square on each side we have a family of rectangles it is possible to get additional information. This is well illustrated in our first theorem.

Theorem 1. *The homology axis of ABC and $G^A(\lambda)G^B(\lambda)G^C(\lambda)$ envelopes the Kiepert parabola of ABC.*

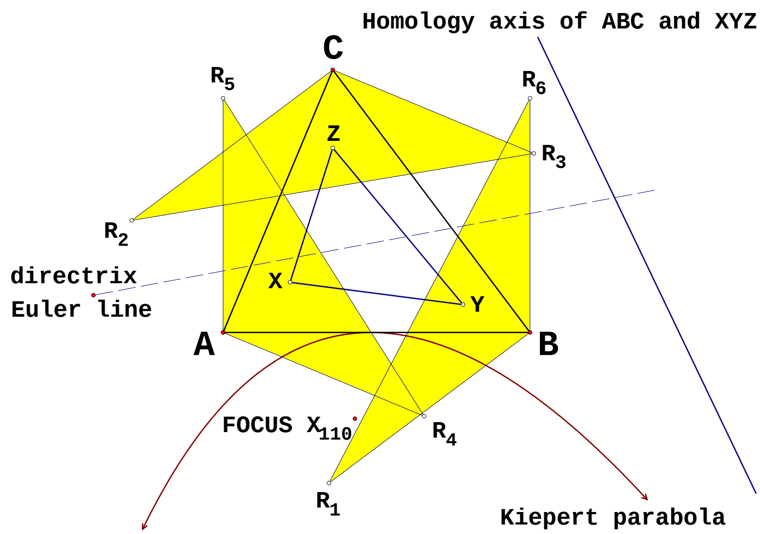

Figure 2. The homology axis of ABC and XYZ envelopes the Kiepert parabola of ABC.

Proof. In our proofs we shall use trilinear coordinates. The advantage of their use is that a high degree of symmetry is present so that it usually suffices to describe part of the information and the rest is self evident. For example, when we write $X_1(1)$ or $I(1)$ or simply say I is 1 this indicates that the incenter has trilinear coordinates $1 : 1 : 1$. We give only the first coordinate while the other two are cyclic permutations of the first. Similarly, $X_2(\frac{1}{a})$, or $G(\frac{1}{a})$, says that the centroid has has trilinears $\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$, where a, b, c are the lengths of the sides of ABC. The expressions in terms of sides a, b, c can be shortened using the following notation.

$$d_a = b - c, \quad d_b = c - a, \quad d_c = a - b, \quad z_a = b + c, \quad z_b = c + a, \quad z_c = a + b,$$
$$t = a + b + c, \quad t_a = b + c - a, \quad t_b = c + a - b, \quad t_c = a + b - c,$$
$$m = abc, \quad m_a = bc, \quad m_b = ca, \quad m_c = ab, \quad T = \sqrt{t t_a t_b t_c},$$

For an integer n, let $t_n = a^n + b^n + c^n$ and $d_{na} = b^n - c^n$, and similarly for other cases. Instead of t_2, t_{2a}, t_{2b}, and t_{2c} we write k, k_a, k_b, and k_c.

In order to achieve even greater economy in our presentation, we shall describe coordinates or equations of only one object from triples of related objects and use cyclic permutations φ and ψ below to obtain the rest. For example, the first vertex A_a of the anticomplementary triangle $A_aB_aC_a$ of ABC has trilinears $-\frac{1}{a} : \frac{1}{b} : \frac{1}{c}$. Then the trilinears of B_a and C_a need not be described because they are easily figured out and memorized by relations $B_a = \varphi(A_a)$ and $C_a = \psi(A_a)$. One must remember always that transformations φ and ψ are not only permutations of letters but also of positions, *i.e.*,

$$\varphi : \ a, \, b, \, c, \, 1, \, 2, \, 3 \mapsto b, \, c, \, a, \, 2, \, 3, \, 1$$

and

$$\psi : \ a, \, b, \, c, \, 1, \, 2, \, 3 \mapsto c, \, a, \, b, \, 3, \, 1, \, 2.$$

Therefore, the trilinears of B_a and C_a are $\frac{1}{a} : -\frac{1}{b} : \frac{1}{c}$ and $\frac{1}{a} : \frac{1}{b} : -\frac{1}{c}$.

The trilinears of the points R_1 and R_2 are equal to $-2\lambda m : c(T + \lambda k_c) : \lambda b k_b$ and $-2\lambda m : \lambda c k_c : b(T + \lambda k_b)$ (while $R_3 = \varphi(R_1)$, $R_4 = \varphi(R_2)$, $R_5 = \psi(R_1)$, and $R_6 = \psi(R_2)$). It follows that the centroid $X_2^A(\lambda)$ or $G^A(\lambda)$ of the triangle AR_4R_5 is $\frac{3T+2a^2\lambda}{-a} : \frac{k_c\lambda}{b} : \frac{k_b\lambda}{c}$.

Hence, the line $G^B(\lambda)G^C(\lambda)$ has equation

$$a(T\,\lambda^2 + 6\,z_{2a}\,\lambda + 9\,T)\,x + b\,\lambda(T\,\lambda + 3\,k_c)\,y + c\,\lambda(T\,\lambda + 3\,k_b)\,z = 0.$$

It intersects the line BC whose equation is $x = 0$ in the point $0 : \frac{T\,\lambda + 3\,k_b}{b} : \frac{T\,\lambda + 3\,k_c}{-c}$. Joining this point with its related points on lines CA and/or AB we get the homology axis of triangles ABC and $G^A(\lambda)G^B(\lambda)G^C(\lambda)$ whose equation is

$$\sum a(T^2\,\lambda^2 + 6\,a^2\,T\,\lambda + 9\,k_b\,k_c)\,x = 0.$$

When we differentiate this equation with respect to λ and solve for λ we get $\lambda = \frac{-3\,(\sum\,a^3\,x)}{T\,(\sum\,a\,x)}$. Substituting this value back into the above equation of the axis we obtain the equation

$$\sum (a^2\,d_{2a}^2\,x^2 - 2\,m_a\,d_{2b}\,d_{2c}\,y\,z) = 0$$

of their envelope. It is well-known (see [1]) that this is in fact the equation of the Kiepert parabola of ABC. □

Recall that triangles ABC and XYZ are *orthologic* provided the perpendiculars from the vertices of ABC to the sides YZ, ZX, and XY of XYZ are concurrent. The point of concurrence of these perpendiculars is denoted by $[ABC,\, XYZ]$. It is well-known that the relation of orthology for triangles is reflexive and symmetric.

Hence, the perpendiculars from the vertices of XYZ to the sides BC, CA, and AB of ABC are concurrent at a point $[XYZ, ABC]$.

Since G (the centroid) befriends H (the orthocenter) it is clear that triangles τ and σ_G^λ are orthologic and $[\sigma_G^\lambda, \tau] = H$. Our next result shows that point $[\tau, \sigma_G^\lambda]$ traces the Kiepert hyperbola of τ.

Theorem 2. *The locus of the orthology center $[\tau, \sigma_G^\lambda]$ of τ and σ_G^λ is the Kiepert hyperbola of ABC.*

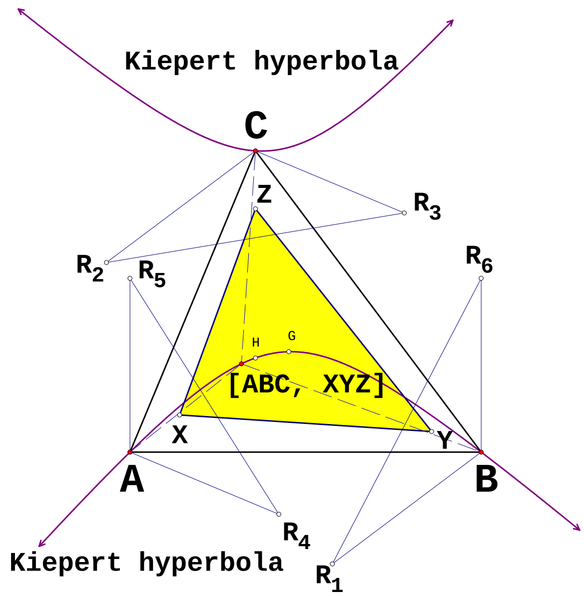

Figure 3. The orthology center $[ABC, XYZ]$ of triangles $\tau = ABC$ and $\sigma_G^\lambda = XYZ$ traces the Kiepert hyperbola of ABC.

Proof. The perpendicular from A onto the line $G^B(\lambda)G^C(\lambda)$ has equation

$$b(T\,\lambda + 3\,k_b)y - c(T\,\lambda + 3\,k_c)z = 0.$$

It follows that $[\tau, \sigma_G^\lambda]$ is $\frac{1}{a(T\lambda+3k_a)}$. This point traces the conic with equation $\sum m_a\, d_{2a}\, y\, z = 0$. The verification that this is the Kiepert hyperbola is easy because we must only check that it goes through A, B, C, $H(\frac{1}{a\,k_a})$, and $G(\frac{1}{a})$. $\square$

Theorem 3. *For every $\lambda \in \mathbb{R}$, the triangles τ and σ_O^λ are homothetic, with center of homothety at the symmedian point K. Hence, they are homologic with homology center K and their homology axis is the line at infinity.*

Proof. The point $\frac{T+z_{2a}\,\lambda}{-abc} : \frac{\lambda}{c} : \frac{\lambda}{b}$ is the circumcenter $O^A(\lambda)$ of the flank AR_4R_5. Since the determinant

$$\begin{vmatrix} 1 & 0 & 0 \\ a & b & c \\ \frac{T+z_{2a}\,\lambda}{-abc} & \frac{\lambda}{c} & \frac{\lambda}{b} \end{vmatrix}$$

is obviously zero, we conclude that the points A, K, and $O^A(\lambda)$ are collinear. In a similar way it follows that $\{B,\, K,\, O^B(\lambda)\}$ and $\{C,\, K,\, O^C(\lambda)\}$ are triples of collinear points. Hence, $\tau \overset{K}{\bowtie} \sigma_O^\lambda$. For $\lambda = -\frac{T}{k}$, the points $O^A(\lambda)$, $O^B(\lambda)$, and $O^C(\lambda)$ coincide with the symmedian point K. For $\lambda \neq -\frac{T}{k}$, the line $O^B(\lambda)O^C(\lambda)$ has equation $(a^2\,\lambda + T)x + \lambda\, a\, b\, y + \lambda\, c\, a\, z = 0$ and is therefore parallel to the sideline BC. Hence, the triangles τ and σ_A^λ are homothetic and the center of this homothety is the symmedian point K of τ. □

Theorem 4. *For every $\lambda \in \mathbb{R}$, the triangles τ and σ_O^λ are orthologic. The orthology center $[\,\tau,\, \sigma_O^\lambda\,]$ is the orthocenter H while the orthology center $[\,\sigma_O^\lambda,\, \tau\,]$ traces the line HK joining the orthocenter with the symmedian point.*

Proof. Since the triangles τ and σ_O^λ are homothetic and their center of similitude is the symmedian point K, it follows that τ and σ_O^λ are orthologic and that $[\,\tau,\, \sigma_O^\lambda\,] = H$. On the other hand, the perpendicular $p(O^A(\lambda),\, BC)$ from $O^A(\lambda)$ onto BC has equation

$$\lambda\, a\, d_{2a}\, k_a\, x + b\,(\lambda\, d_{2a}\, k_a - T\, k_b)\, y + c\,(\lambda\, d_{2a}\, k_a + T\, k_c)\, z = 0.$$

It follows that $[\,\sigma_O^\lambda,\, \tau\,]$ (= the intersection of $p(O^A(\lambda),\, BC)$ and $p(O^B(\lambda),\, CA)$) is the point $\frac{T\, k_b\, k_c + (2\, a^6 - z_{2a}\, a^4 - z_{2a}\, d_{2a}^2)\,\lambda}{a}$. This point traces the line with equation $\sum\, a\, d_{2a}\, k_a^2\, x = 0$. One can easily check that the points H and K lie on it. □

Theorem 5. *The homology axis of τ and σ_H^λ envelopes the parabola with directrix the line HK and focus the central point X_{112}.*

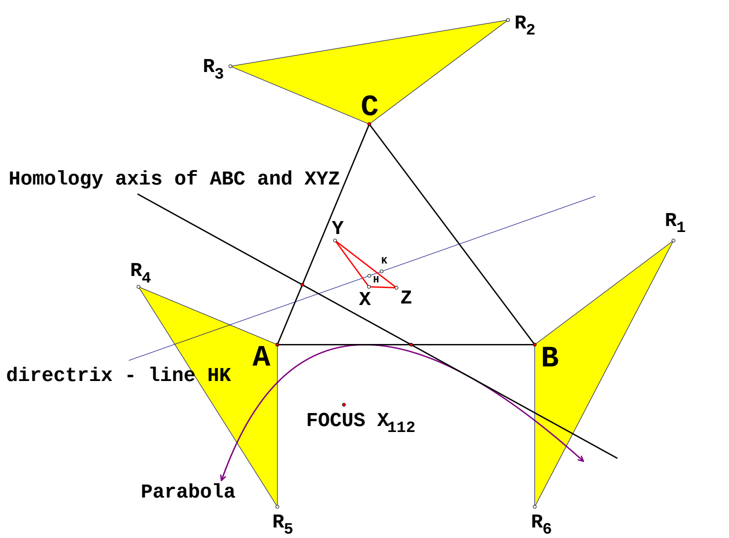

Figure 4. The homology axis of triangles $\sigma_H^\lambda = XYZ$ and $\tau = ABC$ envelopes the parabola with directrix HK and focus X_{112}.

Proof. The orthocenter $H^A(\lambda)$ of the flank AR_4R_5 is $\frac{T-2\lambda k_a}{a\,k_a} : \frac{\lambda}{b} : \frac{\lambda}{c}$. The line $H^B(\lambda)H^C(\lambda)$ has equation

$$a\,(3\,k_b\,k_c\,\lambda^2 - 4\,a^2\,T\,\lambda + T^2)\,x + b\,\lambda\,k_b\,(3\,k_c\,\lambda - T)\,y + c\,\lambda\,k_c\,(3\,k_b\,\lambda - T)\,z = 0.$$

It intersects the sideline BC in the point $0 : \frac{k_c\,(T-3\,k_b\,\lambda)}{b} : \frac{k_b\,(3\,k_c\,\lambda - T)}{c}$. We infer that the homology axis of the triangles τ and σ_H^λ has equation

$$\sum a\,k_a\,(9\,k_b\,k_c\,\lambda^2 - 6\,a^2\,T\,\lambda + T^2)\,x = 0.$$

It envelopes the conic with equation

$$\sum (a^2\,d_{2a}^2\,k_a^2\,x^2 - 2\,m_a\,d_{2b}\,d_{2c}\,k_b\,k_c\,y\,z) = 0.$$

It is easy to check that the above is an equation of a parabola because it intersects the line at infinity $\sum a\,x = 0$ only at the point $\frac{d_{2a}\,k_a}{a}$. On the other hand, we obtain the same equation when we look for the locus of all points P which are at the same distance from the central point $X_{112}(\frac{a}{d_{2a}\,k_a})$ and from the line HK. Hence, the above parabola has the point X_{112} for focus and the line HK for directrix. □

Theorem 6. *For every real number λ the triangles τ and σ_H^λ are orthologic. The locus of the orthology center $[\tau,\,\sigma_H^\lambda]$ is the Kiepert hyperbola of ABC. The locus of the orthology center $[\sigma_H^\lambda,\,\tau]$ is the line HK.*

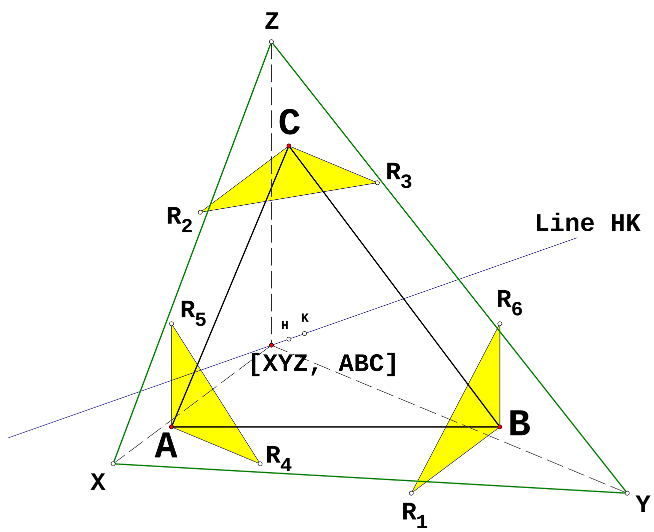

Figure 5. The orthology centers $[\sigma_4^\lambda,\,\tau]$ are on the line HK.

Proof. The perpendicular $p(A,\,H^B(\lambda)H^C(\lambda))$ from A onto the line $H^B(\lambda)H^C(\lambda)$ has equation $M_-(b,\,c)\,y - M_+(c,\,b)\,z = 0$, where

$$M_\pm(b,\,c) = b\,[(3\,a^4 \pm 2\,d_{2a}\,a^2 \pm d_{2a}(b^2 + 3\,c^2))\,\lambda - k_b\,T\,].$$

The lines $p(A,\, H^B(\lambda)H^C(\lambda))$, $p(B,\, H^C(\lambda)H^A(\lambda))$, and $p(C,\, H^A(\lambda)H^B(\lambda))$ concur at the point $\frac{1}{a\,[(a^4+2\,z_{2a}\,a^2-2\,m_{2a}-3\,z_{4a})\,\lambda+k_a\,T\,]}$. Just as in the proof of Theorem 2 we can show that this point traces the Kiepert hyperbola of ABC.

The perpendicular $p(H^A(\lambda),\, BC)$ from $H^A(\lambda)$ onto BC has equation

$$2\,\lambda\,a\,d_{2a}\,k_a\,x+b\,(2\,d_{2a}\,k_a\,\lambda+k_b\,T)\,y+c\,(2\,d_{2a}\,k_a\,\lambda-k_c\,T)\,z=0.$$

The lines $p(H^A(\lambda),\, BC)$, $p(H^B(\lambda),\, CA)$, and $p(H^C(\lambda),\, AB)$ concur at the point $\frac{2\,(2\,a^6-z_{2a}\,a^4-z_{2a}\,d_{2a}^2)\,\lambda-k_b\,k_c\,T}{a}$. We infer that the orthology center $[\sigma_H^\lambda,\, \tau]$ traces the line HK because we get its equation by eliminating the parameter λ from the equations $x=x_0$, $y=y_0$, and $z=z_0$, where x_0, y_0, and z_0 are the trilinears of $[\sigma_H^\lambda,\, \tau]$. □

Theorem 7. *For every $\lambda\in\mathbb{R}\setminus\{0\}$, the triangles ABC and $F^A(\lambda)F^B(\lambda)F^C(\lambda)$ are homologic if and only if the triangle ABC is isosceles.*

Proof. The center $F^A(\lambda)$ of the nine-point circle of the flank AR_4R_5 is

$$\frac{(k_a-a^2)\,\lambda-2\,T}{a}\ :\ \frac{\lambda\,d_{2b}}{-b}\ :\ \frac{\lambda\,d_{2c}}{c}.$$

The line $AF^A(\lambda)$ has equation $b\,d_{2c}\,y+c\,d_{2b}\,z=0$. Hence, the condition for these three lines to concur (expressed in terms of the side lengths) is $2\,m\,d_{2a}\,d_{2b}\,d_{2c}=0$, which immediately implies our claim. □

When triangle ABC is scalene and isosceles, one can show easily that the homology center of ABC and $F^A(\lambda)F^B(\lambda)F^C(\lambda)$ is the midpoint of the base while the homology axis envelopes again the Kiepert parabola of ABC (which agrees with the line parallel to the base through the opposite vertex).

The following two theorems have the same proofs as Theorem 6 and Theorem 1, respectively.

Theorem 8. *For every real number λ the triangles ABC and $F^A(\lambda)F^B(\lambda)F^C(\lambda)$ are orthologic. The orthology centers $[\,\sigma_F^\lambda,\, \tau\,]$ and $[\,\tau,\, \sigma_F^\lambda\,]$ trace the line HK and the Kiepert hyperbola, respectively.*

Theorem 9. *The homology axis of the triangles ABC and $K^A(\lambda)K^B(\lambda)K^C(\lambda)$ envelopes the Kiepert parabola of ABC.*

Theorem 10. *For every $\lambda\in\mathbb{R}\setminus\{0\}$, the triangles ABC and $K^A(\lambda)K^B(\lambda)K^C(\lambda)$ are orthologic if and only if the triangle ABC is isosceles.*

Proof. The symmedian point $K^A(\lambda)$ of the flank AR_4R_5 is

$$\frac{(d_{2a}^2-a^2\,z_{2a})\lambda-T\,(3\,k_a+2\,a^2)}{a}\ :\ \lambda\,b\,k_b:\ \lambda\,c\,k_c.$$

It follows that the perpendicular $p(K^A(\lambda),\, BC)$ from $K^A(\lambda)$ to BC has equation

$$\lambda\,a\,d_{2a}\,T\,x+b\,(\lambda\,d_{2a}\,T-k_b\,(3\,k_a+2\,a^2))\,y+c\,(\lambda\,d_{2a}\,T+k_c\,(3\,k_a+2\,a^2))\,z=0.$$

The triangles ABC and $K^A(\lambda)K^B(\lambda)K^C(\lambda)$ are orthologic if and only if the coefficient determinant of the equations of the lines $p(K^A(\lambda),\ BC)$, $p(K^B(\lambda),\ CA)$, and $p(K^C(\lambda),\ AB)$ is zero. But, this determinant is equal to $-16\lambda m d_{2a} d_{2b} d_{2c} T^6$, which immediately implies that our claim is true. □

When the triangle ABC is scalene and isosceles one can show easily that the orthology centers of ABC and $K^A(\lambda)K^B(\lambda)K^C(\lambda)$ both trace the perpendicular bisector of the base.

The proofs of the following two theorems are left to the reader because they are analogous to proofs of Theorem 1 and Theorem 6, respectively. However, the expressions that appear in them are considerably more complicated.

Theorem 11. *The homology axis of τ and σ_x^λ envelopes the Kiepert parabola of ABC for $x = 15,\ 16,\ 61,\ 62$.*

Theorem 12. *For every real number λ the triangles τ and σ_L^λ are orthologic. The loci of the orthology centers $[\,\tau,\ \sigma_L^\lambda\,]$ and $[\,\sigma_L^\lambda,\ \tau\,]$ are the Kiepert hyperbola and the line HK, respectively.*

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