Forum Geometricorum
Volume 3 (2003) 83–93.

FORUM GEOM
ISSN 1534-1178

Triangle Centers Associated with the Malfatti Circles

Milorad R. Stevanović

Abstract. Various formulae for the radii of the Malfatti circles of a triangle are presented. We also express the radii of the excircles in terms of the radii of the Malfatti circles, and give the coordinates of some interesting triangle centers associated with the Malfatti circles.

1. The radii of the Malfatti circles

The Malfatti circles of a triangle are the three circles inside the triangle, mutually tangent to each other, and each tangent to two sides of the triangle. See Figure 1. Given a triangle ABC, let a, b, c denote the lengths of the sides BC, CA, AB, s the semiperimeter, I the incenter, and r its inradius. The radii of the Malfatti circles of triangle ABC are given by

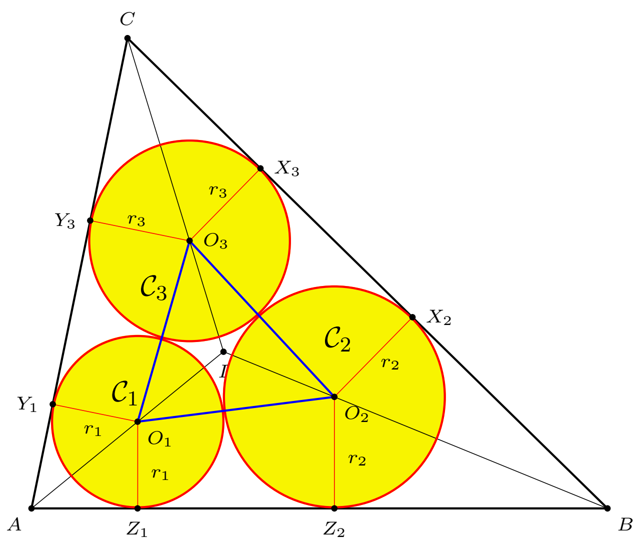

Figure 1

$$\begin{aligned} r_1 &= \frac{r}{2(s-a)}\left(s - r - (IB + IC - IA)\right), \\ r_2 &= \frac{r}{2(s-b)}\left(s - r - (IC + IA - IB)\right), \\ r_3 &= \frac{r}{2(s-c)}\left(s - r - (IA + IB - IC)\right). \end{aligned} \qquad (1)$$

Publication Date: March 24, 2003. Communicating Editor: Paul Yiu.
The author is grateful to the referee and the editor for their valuable comments and helps.

According to F.G.-M. [1, p.729], these results were given by Malfatti himself, and were published in [7] after his death. See also [6]. Another set of formulae give the same radii in terms of a, b, c and r:

$$\begin{aligned} r_1 =& \frac{(IB + r - (s-b))(IC + r - (s-c))}{2(IA + r - (s-a))}, \\ r_2 =& \frac{(IC + r - (s-c))(IA + r - (s-a))}{2(IB + r - (s-b))}, \\ r_3 =& \frac{(IA + r - (s-a))(IB + r - (s-b))}{2(IC + r - (s-c))}. \end{aligned} \tag{2}$$

These easily follow from (1) and the following formulae that express the radii r_1, r_2, r_3 in terms of r and trigonometric functions:

$$\begin{aligned} r_1 =& \frac{\left(1 + \tan\frac{B}{4}\right)\left(1 + \tan\frac{C}{4}\right)}{1 + \tan\frac{A}{4}} \cdot \frac{r}{2}, \\ r_2 =& \frac{\left(1 + \tan\frac{C}{4}\right)\left(1 + \tan\frac{A}{4}\right)}{1 + \tan\frac{B}{4}} \cdot \frac{r}{2}, \\ r_3 =& \frac{\left(1 + \tan\frac{A}{4}\right)\left(1 + \tan\frac{B}{4}\right)}{1 + \tan\frac{C}{4}} \cdot \frac{r}{2}. \end{aligned} \tag{3}$$

These can be found in [10]. They can be used to obtain the following formula which is given in [2, pp.103–106]. See also [12].

$$\frac{2}{r} = \frac{1}{\sqrt{r_1 r_2}} + \frac{1}{\sqrt{r_2 r_3}} + \frac{1}{\sqrt{r_1 r_2}} - \sqrt{\frac{1}{r_1 r_2} + \frac{1}{r_2 r_3} + \frac{1}{r_3 r_1}}. \tag{4}$$

2. Exradii in terms of Malfatti radii

Antreas P. Hatzipolakis [3] asked for the exradii r_a, r_b, r_c of triangle ABC in terms of the Malfatti radii r_1, r_2, r_3 and the inradius r.

Proposition 1.

$$\begin{aligned} r_a - r_1 =& \frac{\frac{2}{r} - \frac{1}{\sqrt{r_2 r_3}}}{\left(\frac{2}{r} - \frac{1}{\sqrt{r_3 r_1}}\right)\left(\frac{2}{r} - \frac{1}{\sqrt{r_1 r_2}}\right)}, \\ r_b - r_2 =& \frac{\frac{2}{r} - \frac{1}{\sqrt{r_3 r_1}}}{\left(\frac{2}{r} - \frac{1}{\sqrt{r_1 r_2}}\right)\left(\frac{2}{r} - \frac{1}{\sqrt{r_2 r_3}}\right)}, \\ r_c - r_3 =& \frac{\frac{2}{r} - \frac{1}{\sqrt{r_1 r_2}}}{\left(\frac{2}{r} - \frac{1}{\sqrt{r_2 r_3}}\right)\left(\frac{2}{r} - \frac{1}{\sqrt{r_3 r_1}}\right)}. \end{aligned} \tag{5}$$

Proof. For convenience we write

$$t_1 := \tan\frac{A}{4}, \qquad t_2 := \tan\frac{B}{4}, \qquad t_3 := \tan\frac{C}{4}.$$

Note that from $\tan\left(\frac{A}{4}+\frac{B}{4}+\frac{C}{4}\right)=1$, we have

$$1-t_1-t_2-t_3-t_1t_2-t_2t_3-t_3t_1+t_1t_2t_3=0. \tag{6}$$

From (3) we obtain

$$\begin{aligned}\frac{2}{r}-\frac{1}{\sqrt{r_2r_3}} &= \frac{t_1}{1+t_1}\cdot\frac{2}{r},\\ \frac{2}{r}-\frac{1}{\sqrt{r_3r_1}} &= \frac{t_2}{1+t_2}\cdot\frac{2}{r},\\ \frac{2}{r}-\frac{1}{\sqrt{r_1r_2}} &= \frac{t_3}{1+t_3}\cdot\frac{2}{r}.\end{aligned} \tag{7}$$

For the exradius r_a, we have

$$r_a=\frac{s}{s-a}\cdot r=\cot\frac{B}{2}\cot\frac{C}{2}\cdot r=\frac{(1-t_2^2)(1-t_3^2)}{4t_2t_3}\cdot r.$$

It follows that

$$\begin{aligned}r_a-r_1 =&(1+t_2)(1+t_3)\cdot\frac{r}{2}\left(\frac{(1-t_2)(1-t_3)}{2t_2t_3}-\frac{1}{1+t_1}\right)\\ =&(1+t_2)(1+t_3)\cdot\frac{r}{2}\cdot\frac{(1+t_1)(1-t_2)(1-t_3)-2t_2t_3}{2t_2t_3(1+t_1)}\\ =&(1+t_2)(1+t_3)\cdot\frac{r}{2}\cdot\frac{2t_1}{2t_2t_3(1+t_1)} \qquad \text{(from (6))}\\ =&\frac{t_1}{1+t_1}\cdot\frac{1+t_2}{t_2}\cdot\frac{1+t_3}{t_3}\cdot\frac{r}{2}.\end{aligned}$$

Now the result follows from (7). □

Note that with the help of (4), the exradii r_a, r_b, r_c can be explicitly written in terms of the Malfatti radii r_1, r_2, r_3. We present another formula useful in the next sections in the organization of coordinates of triangle centers.

Proposition 2.

$$\frac{1}{r_1}-\frac{1}{r_a}=\frac{a}{rs}\cdot\frac{(1+\cos\frac{B}{2})(1+\cos\frac{C}{2})}{1+\cos\frac{A}{2}}.$$

3. Triangle centers associated with the Malfatti circles

Let A' be the point of tangency of the Malfatti circles $\mathcal{C}_2$ and $\mathcal{C}_3$. Similarly define B' and C'. It is known ([4, p.97]) that triangle $A'B'C'$ is perspective with ABC at the *first Ajima-Malfatti point* X_{179}. See Figure 3. We work out the details here and construct a few more triangle centers associated with the Malfatti circles. In particular, we find two new triangle centers P_+ and P_- which divide the incenter I and the first Ajima-Malfatti point harmonically.

3.1. *The centers of the Malfatti circles.* We begin with the coordinates of the centers of the Malfatti circles.

Since O_1 divides the segment AI_a in the ratio $AO_1 : O_1I_a = r_1 : r_a - r_1$, we have $\frac{O_1}{r_1} = \left(\frac{1}{r_1} - \frac{1}{r_a}\right) A + \frac{1}{r_a} \cdot I_A$. With $r_a = \frac{rs}{s-a}$ we rewrite the absolute barycentric coordinates of O_1, along with those of O_2 and O_3, as follows.

$$\begin{aligned} \frac{O_1}{r_1} &= \left(\frac{1}{r_1} - \frac{1}{r_a}\right) A + \frac{s-a}{rs} \cdot I_a, \\ \frac{O_2}{r_2} &= \left(\frac{1}{r_2} - \frac{1}{r_b}\right) B + \frac{s-b}{rs} \cdot I_b, \\ \frac{O_3}{r_3} &= \left(\frac{1}{r_3} - \frac{1}{r_c}\right) C + \frac{s-c}{rs} \cdot I_c. \end{aligned} \tag{8}$$

From these expressions we have, in homogeneous barycentric coordinates,

$$\begin{aligned} O_1 &= \left(2rs\left(\frac{1}{r_1} - \frac{1}{r_a}\right) - a : b : c\right), \\ O_2 &= \left(a : 2rs\left(\frac{1}{r_2} - \frac{1}{r_b}\right) - b : c\right), \\ O_3 &= \left(a : b : 2rs\left(\frac{1}{r_3} - \frac{1}{r_c}\right) - c\right). \end{aligned}$$

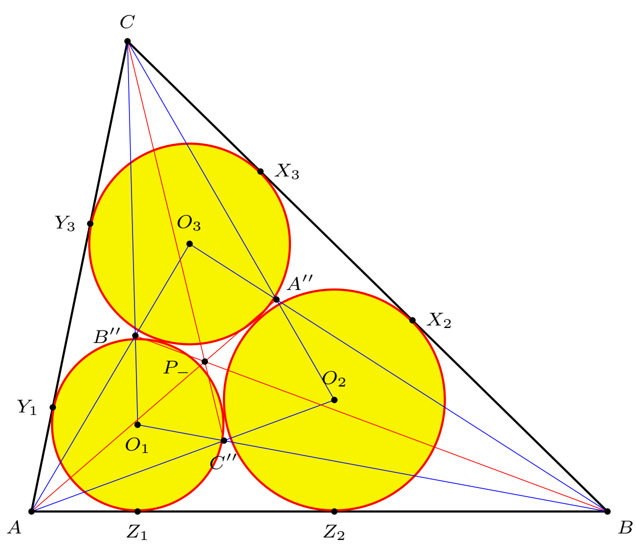

Figure 2

3.2. *The triangle center P_-.* It is clear that $O_1O_2O_3$ is perspective with ABC at the incenter $(a : b : c)$. However, it also follows that if we consider

$$A'' = BO_3 \cap CO_2, \quad B'' = CO_1 \cap AO_3, \quad C'' = AO_2 \cap BO_1,$$

then triangle $A''B''C''$ is perspective with ABC at

$$\begin{aligned} P_- &= \left(2rs\left(\frac{1}{r_1} - \frac{1}{r_a}\right) - a : 2rs\left(\frac{1}{r_2} - \frac{1}{r_b}\right) - b : 2rs\left(\frac{1}{r_3} - \frac{1}{r_c}\right) - c\right) \\ &= \left(\frac{1}{r_1} - \frac{1}{r_a} - \frac{a}{2rs} : \frac{1}{r_2} - \frac{1}{r_b} - \frac{b}{2rs} : \frac{1}{r_3} - \frac{1}{r_c} - \frac{c}{2rs}\right) \\ &= \left(a\left(\frac{(1+\cos\frac{B}{2})(1+\cos\frac{C}{2})}{1+\cos\frac{A}{2}} - \frac{1}{2}\right) : \cdots : \cdots\right) \end{aligned} \tag{9}$$

by Proposition 2. See Figure 2.

Remark. The point P_- appears in [5] as the *first Malfatti-Rabinowitz point* X_{1142}.

3.3. *The first Ajima-Malfatti point.* For the points of tangency of the Malfatti circles, note that A' divides O_2O_3 in the ratio $O_2A' : A'O_3 = r_2 : r_3$. We have, in absolute barycentric coordinates,

$$\left(\frac{1}{r_2} + \frac{1}{r_3}\right)A' = \frac{O_2}{r_2} + \frac{O_3}{r_3} = \frac{a}{rs}\cdot A + \left(\frac{1}{r_2} - \frac{1}{r_b}\right)B + \left(\frac{1}{r_3} - \frac{1}{r_c}\right)C;$$

similarly for B' and C'. In homogeneous coordinates,

$$\begin{aligned} A' &= \left(\frac{a}{rs} : \frac{1}{r_2} - \frac{1}{r_b} : \frac{1}{r_3} - \frac{1}{r_c}\right), \\ B' &= \left(\frac{1}{r_1} - \frac{1}{r_a} : \frac{b}{rs} : \frac{1}{r_3} - \frac{1}{r_c}\right), \\ C' &= \left(\frac{1}{r_1} - \frac{1}{r_a} : \frac{1}{r_2} - \frac{1}{r_b} : \frac{c}{rs}\right). \end{aligned} \tag{10}$$

From these, it is clear that $A'B'C'$ is perspective with ABC at

$$\begin{aligned} P &= \left(\frac{1}{r_1} - \frac{1}{r_a} : \frac{1}{r_2} - \frac{1}{r_b} : \frac{1}{r_3} - \frac{1}{r_c}\right) \\ &= \left(\frac{a(1+\cos\frac{B}{2})(1+\cos\frac{C}{2})}{1+\cos\frac{A}{2}} : \frac{b(1+\cos\frac{C}{2})(1+\cos\frac{A}{2})}{1+\cos\frac{B}{2}}\right. \\ &\qquad \left. : \frac{c(1+\cos\frac{A}{2})(1+\cos\frac{B}{2})}{1+\cos\frac{C}{2}}\right) \\ &= \left(\frac{a}{\left(1+\cos\frac{A}{2}\right)^2} : \frac{b}{\left(1+\cos\frac{B}{2}\right)^2} : \frac{c}{\left(1+\cos\frac{C}{2}\right)^2}\right) \end{aligned} \tag{11}$$

by Proposition 2. The point P appears as X_{179} in [4, p.97], with trilinear coordinates

$$\left(\sec^4 \frac{A}{4} : \sec^4 \frac{B}{4} : \sec^4 \frac{C}{4}\right)$$

computed by Peter Yff, and is named the *first Ajima-Malfatti point*. See Figure 3.

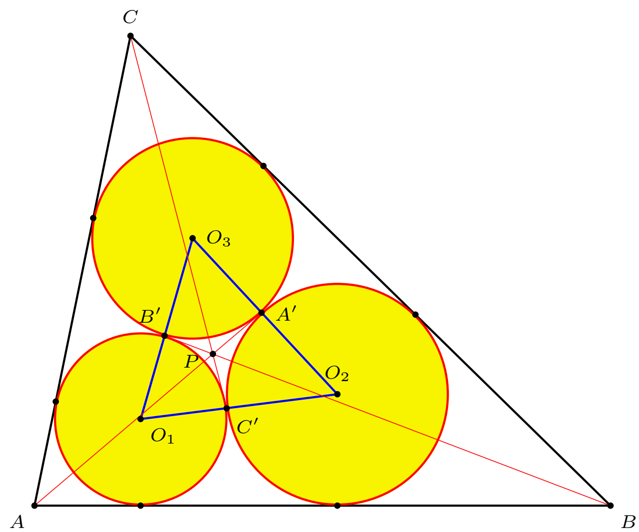

Figure 3

3.4. *The triangle center* P_+. Note that the circle through A', B', C' is orthogonal to the Malfatti circles. It is the radical circle of the Malfatti circles, and is the incircle of $O_1O_2O_3$. The lines O_1A', O_2B', O_3C' are concurrent at the Gergonne point of triangle $O_1O_2O_3$. See Figure 4. As such, this is the point P_+ given by

$$\begin{aligned}
&\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)P_+ = \frac{O_1}{r_1}+\frac{O_2}{r_2}+\frac{O_3}{r_3} \\
=&\left(\frac{1}{r_1}-\frac{1}{r_a}\right)A+\frac{I_a}{r_a}+\left(\frac{1}{r_2}-\frac{1}{r_b}\right)B+\frac{I_b}{r_b}+\left(\frac{1}{r_3}-\frac{1}{r_c}\right)C+\frac{I_c}{r_c} \\
=&\left(\frac{1}{r_1}-\frac{1}{r_a}\right)A+\left(\frac{1}{r_2}-\frac{1}{r_b}\right)B+\left(\frac{1}{r_3}-\frac{1}{r_c}\right)C+\frac{I}{r} \\
=&\left(\frac{1}{r_1}-\frac{1}{r_a}\right)A+\left(\frac{1}{r_2}-\frac{1}{r_b}\right)B+\left(\frac{1}{r_3}-\frac{1}{r_c}\right)C+\frac{1}{2rs}(aA+bB+cC) \\
=&\left(\frac{1}{r_1}-\frac{1}{r_a}+\frac{a}{2rs}\right)A+\left(\frac{1}{r_2}-\frac{1}{r_b}+\frac{b}{2rs}\right)B+\left(\frac{1}{r_3}-\frac{1}{r_c}+\frac{c}{2rs}\right)C.
\end{aligned}$$

It follows that in homogeneous coordinates,

$$P_+ = \left(\frac{1}{r_1} - \frac{1}{r_a} + \frac{a}{2rs} : \frac{1}{r_2} - \frac{1}{r_b} + \frac{b}{2rs} : \frac{1}{r_3} - \frac{1}{r_c} + \frac{c}{2rs}\right)$$
$$= \left(a\left(\frac{(1+\cos\frac{B}{2})(1+\cos\frac{C}{2})}{1+\cos\frac{A}{2}} + \frac{1}{2}\right) : \cdots : \cdots\right) \tag{12}$$

by Proposition 2.

Proposition 3. *The points P_+ and P_- divide the segment IP harmonically.*

Proof. This follows from their coordinates given in (12), (9), and (11). □

From the coordinates of P, P_+ and P_-, it is easy to see that P_+ and P_- divide the segment IP harmonically.

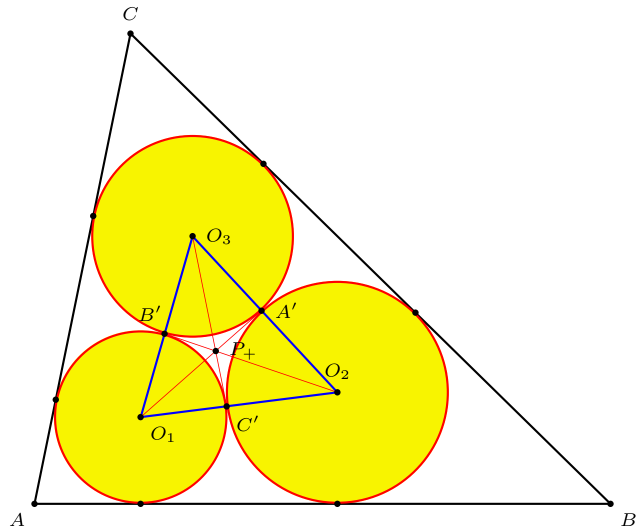

Figure 4

3.5. *The triangle center Q.* Let the Malfatti circle $\mathcal{C}_1$ touch the sides CA and AB at Y_1 and Z_1 respectively. Likewise, let $\mathcal{C}_2$ touch AB and BC at Z_2 and X_2, $\mathcal{C}_3$ touch BC and CA at X_3 and Y_3 respectively. Denote by X, Y, Z the midpoints of the segments X_2X_3, Y_3Y_1, Z_1Z_2 respectively. Stanley Rabinowitz [9] asked if the lines AX, BY, CZ are concurrent. We answer this in the affirmative.

Proposition 4. *The lines AX, BY, CZ are concurrent at a point Q with homogeneous barycentric coordinates*

$$\left(\tan\frac{A}{4} : \tan\frac{B}{4} : \tan\frac{C}{4}\right). \tag{13}$$

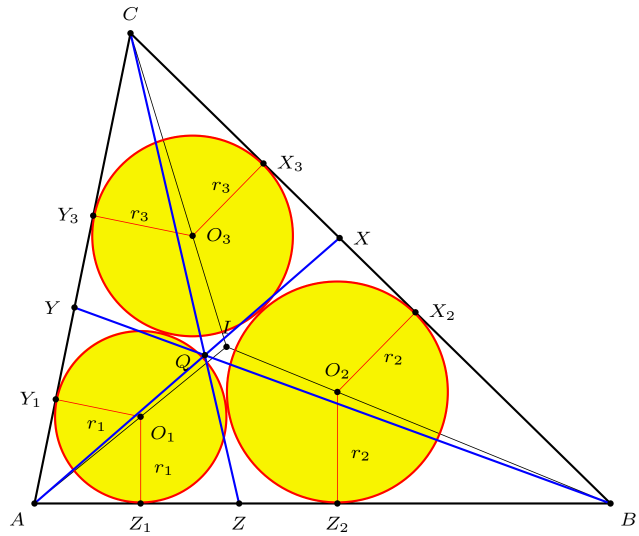

Figure 5

Proof. In Figure 5, we have

$$\begin{aligned} BX =& \frac{1}{2}(a + BX_2 - X_3C) \\ =& \frac{1}{2}\left(a + \frac{r_2}{r}(s-b) - \frac{r_3}{r}(s-c)\right) \\ =& \frac{1}{2}(a + IB - IC) \qquad \text{(from (1))} \\ =& \frac{1}{2}\left(2R\sin A + \frac{r}{\sin\frac{B}{2}} - \frac{r}{\sin\frac{C}{2}}\right) \\ =& 4R\sin\frac{A}{2}\cos\frac{B}{4}\sin\frac{C}{4}\cos\frac{B+C}{4} \end{aligned}$$

by making use of the formula

$$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}.$$

Similarly,

$$XC = \frac{1}{2}(a - BX_2 + X_3C) = 4R\sin\frac{A}{2}\sin\frac{B}{4}\cos\frac{C}{4}\cos\frac{B+C}{4}.$$

It follows that

$$\frac{BX}{XC} = \frac{\cos\frac{B}{4}\sin\frac{C}{4}}{\sin\frac{B}{4}\cos\frac{C}{4}} = \frac{\tan\frac{C}{4}}{\tan\frac{B}{4}}.$$

Likewise,

$$\frac{CY}{YA} = \frac{\tan\frac{A}{4}}{\tan\frac{C}{4}} \quad \text{and} \quad \frac{AZ}{ZB} = \frac{\tan\frac{B}{4}}{\tan\frac{A}{4}},$$

and it follows from Ceva's theorem that AX, BY, CZ are concurrent since

$$\frac{BX}{XC}\cdot\frac{CY}{YA}\cdot\frac{AZ}{ZB}=1.$$

In fact, we can easily identify the homogeneous barycentric coordinates of the intersection Q as given in (13) above since those of X, Y, Z are

$$X=\left(0:\tan\frac{B}{4}:\tan\frac{C}{4}\right),$$
$$Y=\left(\tan\frac{A}{4}:0:\tan\frac{C}{4}\right),$$
$$Z=\left(\tan\frac{A}{4}:\tan\frac{B}{4}:0\right).$$

□

Remark. The coordinates of Q can also be written as

$$\left(\frac{\sin\frac{A}{2}}{1+\cos\frac{A}{2}}:\frac{\sin\frac{B}{2}}{1+\cos\frac{B}{2}}:\frac{\sin\frac{C}{2}}{1+\cos\frac{C}{2}}\right)$$

or

$$\left(\frac{a}{(1+\cos\frac{A}{2})\cos\frac{A}{2}}:\frac{b}{(1+\cos\frac{B}{2})\cos\frac{B}{2}}:\frac{c}{(1+\cos\frac{C}{2})\cos\frac{C}{2}}\right).$$

3.6. *The radical center of the Malfatti circles.* Note that the common tangent of $\mathcal{C}_2$ and $\mathcal{C}_3$ at A' passes through X. This means that $A'X$ is perpendicular to O_2O_3 at A'. This line therefore passes through the incenter I' of $O_1O_2O_3$. Now, the homogeneous coordinates of A' and X can be rewritten as

$$A'=\left(\frac{a}{(1+\cos\frac{A}{2})(1+\cos\frac{B}{2})(1+\cos\frac{C}{2})}:\frac{b}{(1+\cos\frac{B}{2})^2}:\frac{c}{(1+\cos\frac{C}{2})^2}\right),$$
$$X=\left(0:\frac{b}{(1+\cos\frac{B}{2})\cos\frac{B}{2}}:\frac{c}{(1+\cos\frac{C}{2})\cos\frac{C}{2}}\right).$$

It is easy to verify that these two points lie on the line

$$\frac{(1+\cos\frac{A}{2})(\cos\frac{B}{2}-\cos\frac{C}{2})}{a}x-\frac{(1+\cos\frac{B}{2})\cos\frac{B}{2}}{b}y+\frac{(1+\cos\frac{C}{2})\cos\frac{C}{2}}{c}z=0,$$

which also contains the point

$$\left(\frac{a}{1+\cos\frac{A}{2}}:\frac{b}{1+\cos\frac{B}{2}}:\frac{c}{1+\cos\frac{C}{2}}\right).$$

Similar calculations show that the latter point also lies on the lines $B'Y$ and $C'Z$. It is therefore the incenter I' of triangle $O_1O_2O_3$. See Figure 6. This point appears in [5] as X_{483} , the radical center of the Malfatti circles.

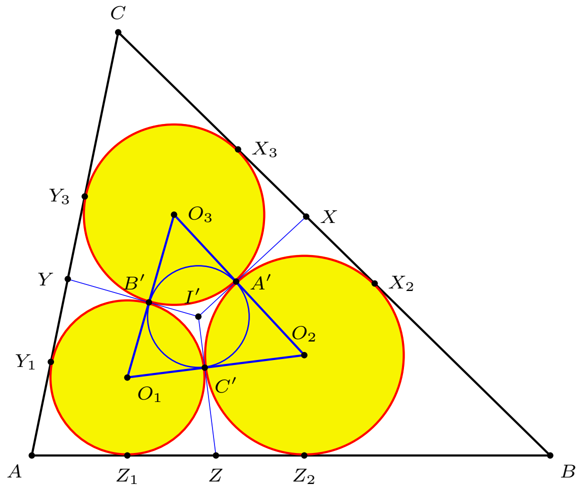

Figure 6

Remarks. (1) The line joining Q and I' has equation

$$\frac{(1+\cos\frac{A}{2})(\cos\frac{B}{2}-\cos\frac{C}{2})}{\sin\frac{A}{2}}x+\frac{(1+\cos\frac{B}{2})(\cos\frac{C}{2}-\cos\frac{A}{2})}{\sin\frac{B}{2}}y$$
$$+\frac{(1+\cos\frac{C}{2})(\cos\frac{A}{2}-\cos\frac{B}{2})}{\sin\frac{C}{2}}z=0.$$

This line clearly contains the point $\left(\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}\right)$, which is the point X_{174}, the Yff center of congruence in [4, pp.94–95].

(2) According to [4], the triangle $A'B'C'$ in §3.3 is also perspective with the excentral triangle. This is because cevian triangles and anticevian triangles are always perspective. The perspector

$$\left(\frac{a\left((2+\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2})^2+\cos\frac{A}{2}(\cos^2\frac{B}{2}+\cos^2\frac{C}{2}-(2+\cos\frac{A}{2})^2)\right)}{1+\cos\frac{A}{2}}:\cdots:\cdots\right)$$

is named the *second Ajima-Malfatti* point X_{180}. For the same reason, the triangle XYZ in §3.5 is also perspective with the excentral triangle. The perspector is the point

$$\left(a\left(-\cos\frac{A}{2}\left(1+\cos\frac{A}{2}\right)+\cos\frac{B}{2}\left(1+\cos\frac{B}{2}\right)+\cos\frac{C}{2}\left(1+\cos\frac{C}{2}\right)\right):\cdots:\cdots\right).$$

This point and the triangle center P_+ apparently do not appear in the current edition of [5].

Editor's endnote. The triangle center Q in §3.5 appears in [5] as the *second Malfatti-Rabinowitz point* X_{1143}. Its coordinates given by the present editor [13] were not correct owing to a mistake in a sign in the calculations. In the notations of [13], if

α, β, γ are such that

$$\sin^2\alpha = \frac{a}{s}, \quad \sin^2\beta = \frac{b}{s}, \quad \sin^2\gamma = \frac{c}{s},$$

and $\lambda = \frac{1}{2}(\alpha+\beta+\gamma)$, then the homogeneous barycentric coordinates of Q are

$$(\cot(\lambda-\alpha) : \cot(\lambda-\beta) : \cot(\lambda-\gamma)).$$

These are equivalent to those given in (13) in simpler form.

References

[1] F. G.-M., *Exercices de Géométrie*, 6th ed., 1920; Gabay reprint, Paris, 1991.
[2] H. Fukagawa and D. Pedoe, *Japanese Temple Geometry Problems*, Charles Babbage Research Centre, Winnipeg, 1989.,p.p.103-106.
[3] A. P. Hatzipolakis, Hyacinthos message 2375, January 8, 2001.
[4] C. Kimberling, Triangle centers and central triangles, *Congressus Numerantium*, 129 (1998) 1 – 285.
[5] C. Kimberling, *Encyclopedia of Triangle Centers*, August 22, 2002 edition, available at http://www2.evansville.edu/ck6/encyclopedia/; February 26, 2003 edition available at http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.
[6] U. J. Knisely, Solution to problem Problem 186. *The Mathematical Visitor*, 1 (1881) 189; reprinted in [8, pp.148–149].
[7] Malfatti, *Annales mathématiques de Gergonne*, 1 (1811) 347.
[8] S. Rabinowitz, *Problems and solutions from the Mathematical Visitor, 1877–1896*, MathPro Press, Westford, MA, 1996.
[9] S. Rabinowitz, Hyacinthos message 4610, December 30, 2001.
[10] E. B. Seitz, Solution to problem 186. *The Mathematical Visitor*, 1 (1881) 190; reprinted in [8, p.150].
[11] M. R. Stevanović, Hyacinthos message 4613, December 31, 2001.
[12] G. Tsintsifas, Solution to problem 618, *Crux Math.*, 8 (1982) 82–85.
[13] P. Yiu, Hyacinthos message 4615, December 30, 2001.

Milorad R. Stevanović: Technical Faculty, Svetog Save 65, 32000 Čačak, Serbia, Yugoslavia
E-mail address: milmath@ptt.yu