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The Parasix Configuration and Orthocorrespondence

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Abstract. We introduce the parasix configuration, which consists of two congruent triangles. The conditions of these triangles to be orthologic with ABC or a circumcevian triangle, to form a cyclic hexagon, to be equilateral or to be degenerate reveal a relation with orthocorrespondence, as defined in [1].

1. The parasix configuration

Consider a triangle ABC of reference with finite points P and Q not on its sidelines. Clark Kimberling [2, §§9.7,8] has drawn attention to configurations defined by six triangles. As an example of such configurations we may create six triangles using the lines ℓ_a, ℓ_b and ℓ_c through Q parallel to sides a, b and c respectively. The triples of lines (ℓ_a, b, c), (a, ℓ_b, c) and (a, b, ℓ_c) bound three triangles which we refer to as the *great paratriple*. Figure 1a shows the A-triangle of the great paratriple. On the other hand, the triples (a, ℓ_b, ℓ_c), (ℓ_a, b, ℓ_c) and (ℓ_a, ℓ_b, c) bound three triangles which we refer to as the *small paratriple*. See Figure 1b.

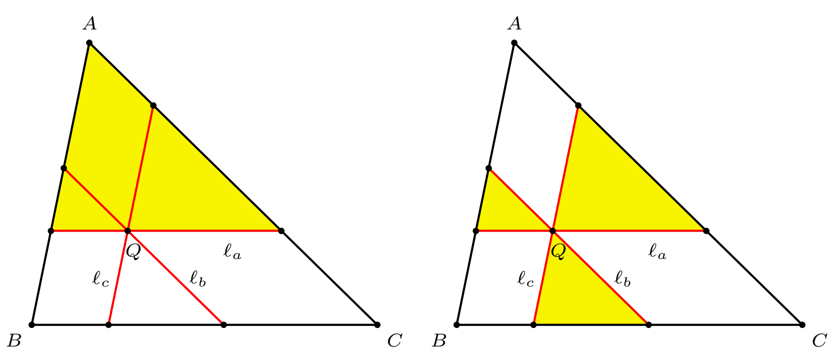

Figure 1a

Figure 1b

Clearly these six triangles are all homothetic to ABC, and it is very easy to find the homothetic images of P in these triangles, A_g in the A-triangle bounded by (ℓ_a, b, c) in the great paratriple, and A_s in the A-triangle bounded by (a, ℓ_b, ℓ_c) in the small paratriple; similarly for B_g, C_g, B_s, C_s. These six points form the *parasix configuration of* P *with respect to* Q, or shortly $\mathsf{Parasix}(P, Q)$. See Figure 2. If in homogeneous barycentric coordinates with reference to ABC, $P = (u : v : w)$ and $Q = (f : g : h)$, then these are the points

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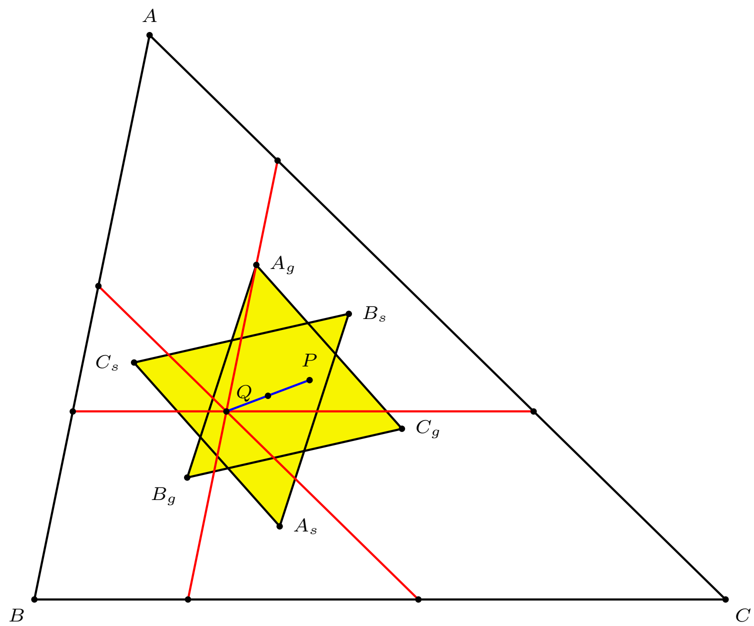

Figure 2. Parasix(P, Q)

$$
\begin{aligned}
A_g =&(u(f+g+h)+f(v+w):v(g+h):w(g+h)),\\
B_g =&(u(f+h):g(u+w)+v(f+g+h):w(f+h)),\\
C_g =&(u(f+g):v(f+g):h(u+v)+w(f+g+h));\\
A_s =&(uf:g(u+w)+v(f+g):h(u+v)+w(f+h)),\\
B_s =&(u(f+g)+f(v+w):vg:h(u+v)+w(g+h)),\\
C_s =&(u(f+h)+f(v+w):g(u+w)+v(g+h):wh).
\end{aligned} \tag{1}
$$

Proposition 1. (1) *Triangles $A_gB_gC_g$ and $A_sB_sC_s$ are symmetric about the midpoint of segment PQ.*
(2) *The six points of a parasix configuration lie on a central conic.*
(3) *The centroids of triangles $A_gB_gC_g$ and $A_sB_sC_s$ trisect the segment PQ.*

Proof. It is clear from the coordinates given above that the segments A_gA_s, B_gB_s, C_gC_s, PQ have a common midpoint

$$(f(u+v+w)+u(f+g+h):\cdots:\cdots).$$

The six points therefore lie on a conic with this common midpoint as center. For (3), it is enough to note that the centroids G_g and G_s of $A_gB_gC_g$ and $A_sB_sC_s$ are the points

$$
\begin{aligned}
G_g =&(2u(f+g+h)+f(u+v+w):\cdots:\cdots),\\
G_s =&(u(f+g+h)+2f(u+v+w):\cdots:\cdots).
\end{aligned}
$$

It follows that vectors $\overrightarrow{PG_g}=\frac{1}{3}\,\overrightarrow{PQ}$ and $\overrightarrow{PG_s}=\frac{2}{3}\,\overrightarrow{PQ}$. □

While $\mathsf{Parasix}(P,Q)$ consists of the two triangles $A_gB_gC_g$ and $A_sB_sC_s$, we write $\widetilde{A}_g\widetilde{B}_g\widetilde{C}_g$ and $\widetilde{A}_s\widetilde{B}_s\widetilde{C}_s$ for the two corresponding triangles of $\mathsf{Parasix}(Q,P)$. From (1) we easily derive their coordinates by interchanging the roles of f, g, h, and u, v, w. Note that $\widetilde{G}_s = G_g$ and $\widetilde{G}_g = G_s$.

Let P_A and Q_A be the the points where AP and AQ meet BC respectively, and let $AP : PP_A = t_P : 1 - t_P$ while $AQ : QQ_A = t_Q : 1 - t_Q$. Then it is easy to see that

$$AA_g : A_gP_A = A\widetilde{A}_g : \widetilde{A}_gQ_A = t_Pt_Q : 1 - t_Pt_Q$$

so that the line $A_g\widetilde{A}_g$ is parallel to BC. By Proposition 1, $A_s\widetilde{A_s}$ is also parallel to BC.

Proposition 2. (a) *The lines $A_g\widetilde{A}_g$, $B_g\widetilde{B}_g$ and $C_g\widetilde{C}_g$ bound a triangle homothetic to ABC. The center of homothety is the point*

$$(f(u+v+w)+u(g+h) : g(u+v+w)+v(h+f) : h(u+v+w)+w(f+g)).$$

The ratio of homothety is

$$-\frac{fu+gv+hw}{(f+g+h)(u+v+w)}.$$

(b) *The lines $A_s\widetilde{A}_s$, $B_s\widetilde{B}_s$ and $C_s\widetilde{C}_s$ bound a triangle homothetic to ABC with center of homothety $(uf : vg : wh)$* [1] *The ratio of homothety is*

$$1-\frac{fu+gv+hw}{(f+g+h)(u+v+w)}.$$

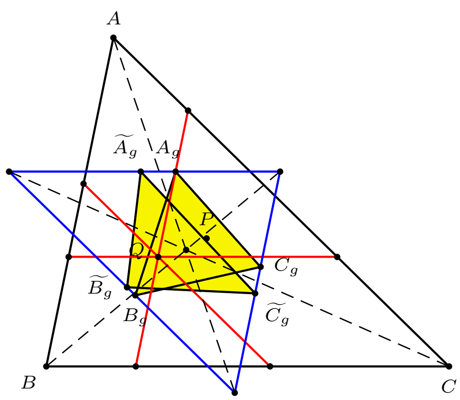

Figure 3a

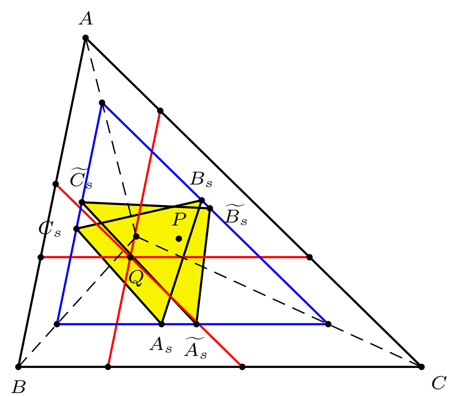

Figure 3b

[1]This point is called the barycentric product of P and Q. Another construction was given by P. Yiu in [4]. These homothetic centers are collinear with the midpoint of PQ.

2. Parasix loci

We present a few line and conic loci associated with parasix configurations. For $P = (u : v : w)$, we denote by

(i) $\mathcal{L}_P$ the trilinear polar of P, which has equation

$$\frac{x}{u} + \frac{y}{v} + \frac{z}{w} = 0;$$

(ii) $\mathcal{C}_P$ the circumconic with perspector P, which has equation

$$\frac{u}{x} + \frac{v}{y} + \frac{w}{z} = 0.$$

2.1. *Area of parasix triangles.* The parasix triangles $A_gB_gC_g$ and $A_sB_sC_s$ have a common area

$$\frac{ghu + hfv + fgw}{(f+g+h)^2(u+v+w)}. \tag{2}$$

Proposition 3. (a) *For a given Q, the locus of P for which the triangles $A_gB_gC_g$ and $A_sB_sC_s$ have a fixed (signed) area is a line parallel to $\mathcal{L}_P$.*

(b) *For a given P, the locus of Q for which the triangles $A_gB_gC_g$ and $A_sB_sC_s$ have a fixed (signed) area is a conic homothetic to $\mathcal{C}_P$ at its center.*

In particular, the parasix triangles degenerate into two parallel lines if and only if

$$\frac{u}{f} + \frac{v}{g} + \frac{w}{h} = 0. \tag{$*$}$$

This condition can be construed in two ways: $P \in \mathcal{L}_Q$, or equivalently, $P \in \mathcal{C}_P$. See §6.

2.2. *Perspectivity with the pedal triangle.*

Proposition 4. (a) *Given P, the locus of Q so that $A_sB_sC_s$ is perspective to the pedal triangle of Q is the line*[2]

$$\sum_{\text{cyclic}} S_A(S_Bv - S_Cw)(-uS_A + vS_B + wS_C)x = 0.$$

This line passes through the orthocenter H and the point

$$\left(\frac{1}{S_A(-uS_A + vS_B + wS_C)} : \cdots : \cdots\right),$$

which can be constructed as the perspector of ABC and the cevian triangle of P in the orthic triangle.

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[2]Here we adopt J.H. Conway's notation by writing S for *twice* of the area of triangle ABC and

$$S_A = S\cdot\cot A = \frac{b^2+c^2-c^2}{2},\ S_B = S\cdot\cot B = \frac{c^2+a^2-b^2}{2},\ S_C = S\cdot\cot C = \frac{a^2+b^2-c^2}{2}.$$

These satisfy $S_{AB} + S_{BC} + S_{CA} = S^2$. The expressions S_{AB}, S_{BC}, S_{CA} stand for S_AS_B, S_BS_C, S_CS_A respectively.

2.3. *Parallelogy.* A triangle is said to be parallelogic to a second triangle if the lines through the vertices of the triangle parallel to the corresponding opposite sides of the second triangle are concurrent.

Proposition 5. (a) *Given $P = (u : v : w)$, the locus of Q for which ABC is parallelogic to $A_gB_gC_g$ (respectively $A_sB_sC_s$) is the line $(v+w)x+(w+u)y+(u+v)z = 0$, which can be constructed as the trilinear polar of the isotomic conjugate of the complement of P.*

(b) *Given $Q = (f : g : h)$, the locus of P for which ABC is parallelogic to $A_gB_gC_g$ (respectively $A_sB_sC_s$) is the line $(g+h)x+(h+f)y+(f+g)z = 0$, which can be constructed as the trilinear polar of the isotomic conjugate of the complement of Q.*

2.4. *Perspectivity with ABC.* Clearly $A_gB_gC_g$ is perspective to ABC at P. The perspectrix is the line $gh(g+h)x+fh(f+h)y+fg(f+g)z = 0$, parallel to the trilinear polar of Q. Given P, the locus of Q such that $A_sB_sC_s$ is perspective to ABC is the cubic

$$(v+w)x(wy^2 - vz^2) + (u+w)y(uz^2 - wx^2) + (u+v)z(vx^2 - uy^2) = 0,$$

which is the isocubic with pivot $(v+w : w+u : u+v)$ and pole P. For $P = K$, the symmedian point, this is the isogonal cubic with pivot $X_{141} = (b^2+c^2 : c^2+a^2 : a^2+b^2)$.

3. Orthology

Some interesting loci associated with the orthology of triangles attracted our attention because of their connection with the orthocorrespondence defined in [1]. We recall that two triangles are orthologic if the perpendiculars from the vertices of one triangle to the opposite sides of the corresponding vertices of the other triangle are concurrent.

First, consider the locus of Q, given P, such that the triangles $A_gB_gC_g$ and $A_sB_sC_s$ are orthologic to ABC. We can find this locus by simple calculation since this is also the locus such that $A_gB_gC_g$ is perspective to the triangle of the infinite points of the altitudes, with coordinates

$$H_A^\infty = (-a^2, S_C, S_B), \quad H_B^\infty = (S_C, -b^2, S_A), \quad H_C^\infty = (S_B, S_A, -c^2).$$

The lines $A_gH_A^\infty$, $V_gH_B^\infty$ and $C_gH_C^\infty$ concur if and only if Q lies on the line

$$(S_Bv - S_Cw)x + (S_Cw - S_Au)y + (S_Au - S_Bv)z = 0, \qquad (3)$$

which is the line through the centroid G and the orthocorrespondent of P, namely, the point [3]

$$P^\perp = (u(-S_Au + S_Bv + S_Cw) + a^2vw : \cdots : \cdots).$$

The line (3) is the orthocorrespondent of the line HP. See [1, §2.4].

[3]The lines perpendicular at P to AP, BP, CP intersect the respective sidelines at three collinear points. The orthocorrespondent of P is the trilinear pole $P^\perp$ of the line containing these three intersections.

For the second locus problem, we let Q be given, and ask for the locus of P such that the triangles $A_gB_gC_g$ and $A_sB_sC_s$ are orthologic to ABC. The computations are similar, and again we find a line as the locus:

$$S_A(g-h)x + S_B(h-f)y + S_C(f-g)z = 0.$$

This is the line through H, and the two anti-orthocorrespondents of Q. See [1, Figure 2]. It is the anti-orthocorrespondent of the line GQ.

Given P, for both $A_gB_gC_g$ and $\widetilde{A}_g\widetilde{B}_g\widetilde{C}_g$ to be orthologic to ABC, the point Q has to be the intersection of the line $GP^{\perp}$ ((3) above) and

$$S_A(v-w)x + S_B(w-u)y + S_C(u-v)z = 0,$$

the anti-orthocorrespondent of GP. This is the point

$$\tau(P) = \big(S_A(c^2-b^2)u^2 + (S_{AC}-S_{BB})uv - (S_{AB}-S_{CC})uw + a^2(c^2-b^2)vw \\ : \cdots : \cdots\big).$$

The point $\tau(P)$ is not well defined if all three coordinates of $\tau(P)$ are equal to zero, which is the case exactly when P is either K, the orthocenter H, or the centroid G. The pre-images of these points are lines: GH (the Euler line), GK, and HK for K, G and H respectively. Outside these lines the mapping $P \longmapsto \tau(P)$ is an involution. Note that P and $\tau(P)$ are collinear with the symmedian point K.

The fixed points of τ are the points of the Kiepert hyperbola

$$(b^2-c^2)yz + (c^2-a^2)xz + (a^2-b^2)xy = 0.$$

More precisely, the line joining $\tau(P)$ to H meets GP on the Kiepert hyperbola. Therefore we may characterize $\tau(P)$ as the intersection of the line PK with the polar of P in the Kiepert hyperbola. [4]

In the table below we give the first coordinates of some well known triangle centers and their images under τ. The indexing of triangle centers follows [3].

P	first coordinate	$\tau(P)$	first coordinate
X_1	a	X_9	$a(s-a)$
X_7	$(s-b)(s-c)$	X_{948}	$(s-b)(s-c)F$
X_8	$s-a$		$a^2+(b+c)^2$
X_{19}			aG
X_{34}			$a(s-b)(s-c)(a^2+(b+c)^2)$
X_{37}		X_{72}	$a(b+c)S_A$
X_{42}	$a^2(b+c)$	X_{71}	$a^2(b+c)S_A$
X_{57}	$a/(s-a)$	X_{223}	$a(s-b)(s-c)F$
X_{58}		X_{572}	a^2G

[4] This is also called the *Hirst inverse* of P with respect to K. See the glossary of [3].

Here,

$$F = a^3 + a^2(b+c) - a(b+c)^2 - (b+c)(b-c)^2,$$
$$G = a^3 + a^2(b+c) + a(b+c)^2 + (b+c)(b-c)^2,$$

We may also wonder, given P outside the circumcircle, for which Q are the $\mathsf{Parasix}(P,Q)$ triangles $A_gB_gC_g$ and $A_sB_sC_s$ orthologic to the circumcevian triangle of P. The A-vertex of the circumcevian triangle of P has coordinates

$$\left(-a^2yz : (b^2z + c^2y)y : (b^2z + c^2y)z\right).$$

Hence we find that the lines from the vertices of the circumcevian triangle of P perpendicular to the corresponding sides of $A_gB_gC_g$ concur if and only if

$$(uyz + vxz + wxy)L = 0, \tag{4}$$

where

$$L = \sum_{\text{cyclic}} (c^2v^2 + 2S_Avw + b^2w^2)((c^2S_Cv - b^2S_Bw)u^2 + a^2((c^2v^2 - b^2w^2)u + (S_Bv - S_Cw)vw))x.$$

The first factor in (4) represents the circumconic with perspector P, and when Q is on this conic, $\mathsf{Parasix}(P,Q)$ is degenerate, see §6 below. The second factor L yields the locus we are looking for, a line passing through $P^\perp$. [5]

A point X lies on the line $L = 0$ if and only if P lies on a bicircular circumquintic through the in- and excenters[6]. For the special case $X = G$ this quintic decomposes into $\mathcal{L}_\infty$ (with multiplicity 2) and the McCay cubic. [7] In other words, for any P on the McCay cubic, the circumcevian triangle of P is orthologic to the $\mathsf{Parasix}(P,Q)$ triangles if and only if Q lies on the line $GP^\perp$.

4. Concyclic Parasix(P,Q)-hexagons

We may ask, given P, for which Q the parasix configuration yields a cyclic hexagon. This is equivalent to the circumcenter of $A_gB_gC_g$ being equal to the midpoint of segment PQ. Now the midpoint of PQ lies on the perpendicular bisector of B_gC_g if and only if Q lies on the line

$$-(w(S_Au + S_Bv - S_Cw) + c^2uv)y + (v(S_Au - S_Bv + S_Cw)v + b^2wu)z = 0,$$

which is indeed the cevian line $AP^\perp$. Remarkably, we find the same cevian line as locus for Q satisfying the condition that $B_gC_g \perp AP$.

Proposition 6. *The following statements are equivalent.*

(1) $\mathsf{Parasix}(P,Q)$ *yields a cyclic hexagon.*

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[5]The line $L = 0$ is not defined when P is an in/excenter. This means that, for any Q, triangles $A_gB_gC_g$ and $A_sB_sC_s$ in $\mathsf{Parasix}(P,Q)$ are orthologic to the circumcevian triangle of P. This is not surprising since P is the orthocenter of its own circumcevian triangle. For $P = X_3$, $L = 0$ is the line GK, while for $P = X_{13}, X_{14}$, it is the parallel at P to the Euler line.

[6]This quintic has equation $Q_Ax + Q_By + Q_Cz = 0$ where Q_A represents the union of the circle center A, radius 0 and the Van Rees focal which is the isogonal pivotal cubic with pivot the infinite point of AH and singular focus A.

[7]The McCay cubic is the isogonal cubic with pivot O given by the equation $\sum_{\text{cyclic}} a^2S_Ax(c^2y^2 - b^2z^2) = 0$.

(2) *$A_gB_gC_g$ and $A_sB_sC_s$ are homothetic to the antipedal triangle of P.*
(3) *Q is the orthocorrespondent of P.*

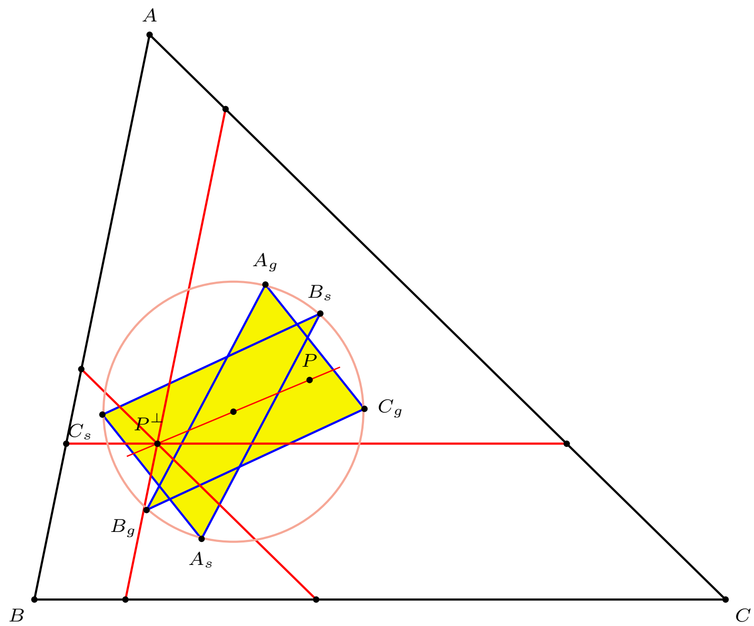

Figure 4

The center of the circle containing the 6 points is the midpoint of PQ.

The homothetic centers and the circumcenter of the cyclic hexagon are collinear.

A nice example is the circle around $\mathsf{Parasix}(H,G)$. It is homothetic to the circumcircle and nine point circle through H with factors $\frac{1}{3}$ and $\frac{2}{3}$ respectively. The center of the circle divides OH in the ratio $2:1$.[8] The antipedal triangle of H is clearly the anticomplementary triangle of ABC. The two homothetic centers divide the same segment in the ratios $5:2$ and $3:2$ respectively.[9] See Figure 5.

As noted in [1], $P = P^{\perp}$ only for the Fermat-Torricelli points X_{13} and X_{14}. The vertices of $\mathsf{parasix}(X_{13}, X_{13})$ and $\mathsf{Parasix}(X_{14}, X_{14})$ form regular hexagons. See Figure 6.

5. Equilateral triangles

The last example raises the question of finding, for given P, the points Q for which the triangles $A_gB_gC_g$ and $A_sB_sC_s$ are equilateral. We find that the A-median of $A_gB_gC_g$ is also an altitude in this triangle if and only if Q lies on the

[8] This is also the midpoint of GH, the center of the orthocentroidal circle, the point X_{381} in [3].

[9] These have homogeneous barycentric coordinates $(3a^4+2a^2(b^2+c^2)-5(b^2-c^2)^2 : \cdots : \cdots)$ and $(a^4-2a^2(b^2+c^2)+3(b^2-c^2)^2 : \cdots : \cdots)$ respectively. They are not in the current edition of [3].

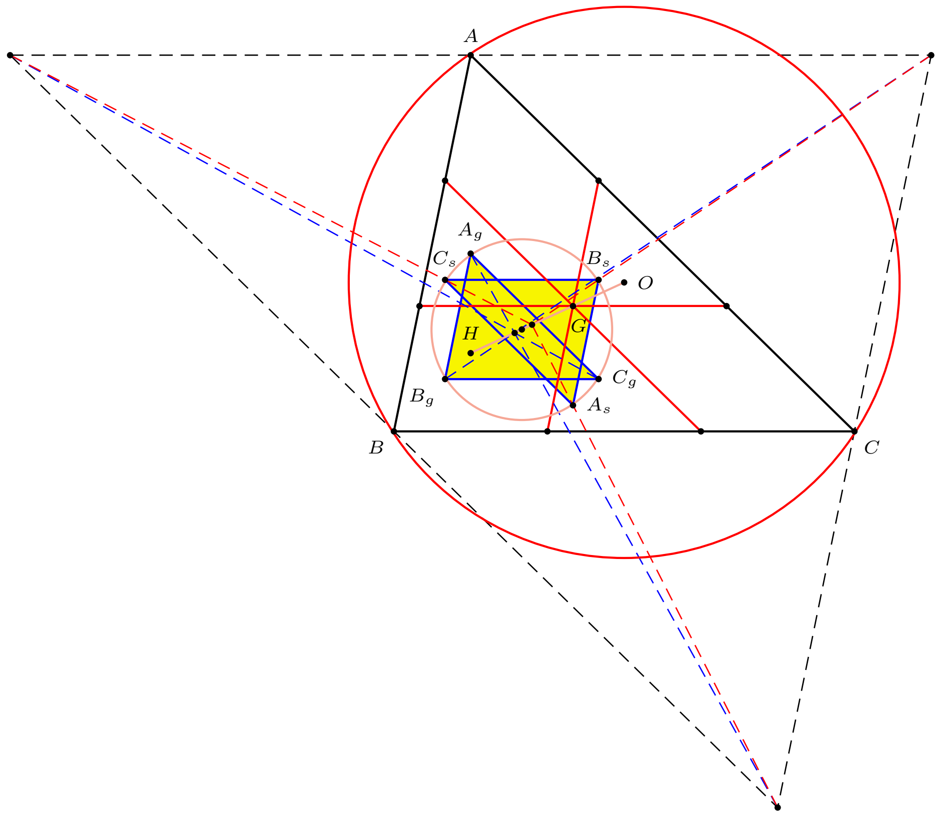

Figure 5. Parasix(H, G)

conic

$$-2((S_Au + S_Bv - S_Cw)w + c^2uv)xy + 2((S_Au - S_Bv + S_Cw)v + b^2uw)xz$$
$$-(c^2u^2 + a^2w^2 + 2S_Buw)y^2 + (b^2u^2 + a^2v^2 + 2S_Cuv)z^2 = 0.$$

We find an analogous conic for the B-median of $A_gB_gC_g$ to be an altitude. The two conics intersect in four points: two imaginary points and the points

$$Q_{1,2} = \left((-S_Au + S_Bv + S_Cw)u + a^2vw \pm \frac{1}{3}\sqrt{3}Su(u + v + w) : \cdots : \cdots\right).$$

Proposition 7. *Given P, there are two (real) points Q for which triangles $A_gB_gC_g$ and $A_sB_sC_s$ are equilateral. These two points divide $PP^{\perp}$ harmonically.*

The points $Q_{1,2}$ from Proposition 7 can be constructed in the following way, using the fact that P, G_s, G_g and $P^{\perp}$ are collinear.

Start with a point G' on $PP^{\perp}$. We shall construct an equilateral triangle $A'B'C'$ with vertices on AP, BP and CP respectively and centroid at G'. This triangle must be homothetic to one of the equilateral triangles $A_gB_gC_g$ of Proposition 7 through P.

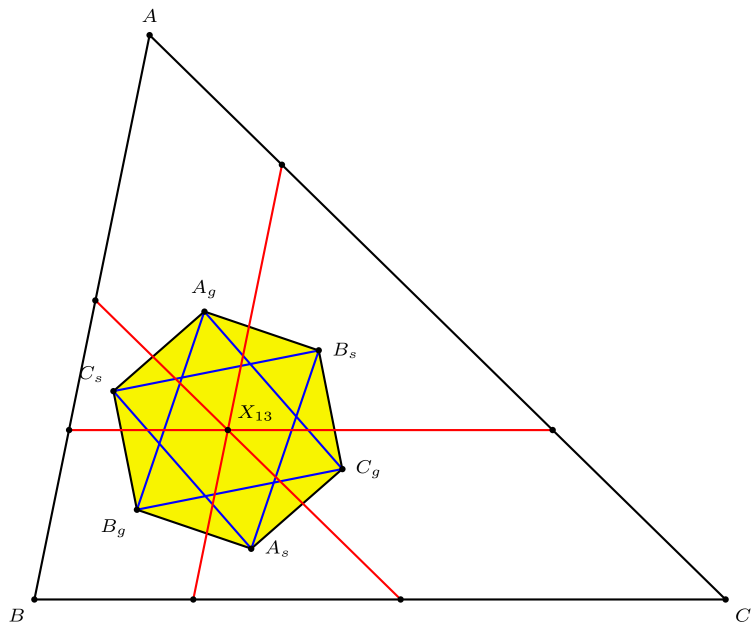

Figure 6. The parasix configuration $\mathsf{Parasix}(X_{13}, X_{13})$

Consider the rotation ρ about G' through $\pm\frac{2\pi}{3}$. The image of AP intersects BP in a point B'. Now let C' be the image of B' and A' the image of C'. Then $A'B'C'$ is equilateral, A' lies on AP, G' is the centroid and C' must lie on CP.

The homothety with center A that maps P to A' also maps BC to a line ℓ_a. Similarly we find ℓ_b and ℓ_c. These lines enclose a triangle $A''B''C''$ homothetic to ABC. We of course want to find the case for which $A''B''C''$ degenerates into one point, which is the Q we are looking for. Since all possible equilateral $A'B'C'$ of the same orientation are homothetic through P, the trianges $A''B''C''$ are all homothetic to ABC through the same point. So the homothety center of $A''B''C''$ and ABC is the point Q we are looking for.

6. Degenerate parasix triangles

We begin with a simple interesting fact.

Proposition 8. *Every line through P intersects the circumconic $\mathcal{C}_P$ at two real points.*

Proof. For the special case of the symmedian point K this is clear, since K is the interior of the circumcircle. Now, there is a homography φ fixing A, B, C and transforming $P = (u : v : w)$ into $K = (a^2 : b^2 : c^2)$. It is given by

$$\varphi(x : y : z) = \left(\frac{a^2}{u}x : \frac{b^2}{v}y : \frac{c^2}{w}z\right),$$

and is a projective transformation mapping $\mathcal{C}_P$ into the circumcircle and any line through P into a line through K. If ℓ is a line through P, then $\varphi(\ell)$ is a line through K, intersecting the circumcircle at two real points q_1 and q_2. The circumcircle and the circumconic $\mathcal{C}_P$ have a fourth real point Z in common, which is the trilinear pole of the line PK. For any point M on $\mathcal{C}_P$, the points Z, M, $\varphi(M)$ are collinear. The second intersections of the lines Zq_1 and Zq_2 are common points of ℓ and the circumconic $\mathcal{C}_P$. □

In §2, we have seen that the parasix triangles are degenerate if and only if $P \in \mathcal{L}_Q$ or equivalently, $Q \in \mathcal{C}_P$. This means that for each line ℓ_P through P intersecting the circumconic $\mathcal{C}_P$ at Q_1 and Q_2, the triangles of $\mathsf{Parasix}(P, Q_i)$, $i = 1, 2$, are degenerate.

Theorem 9. *For $i = 1, 2$, the two lines containing the degenerate triangles of the parasix configuration $\mathsf{Parasix}(P, Q_i)$ are parallel to a tangent from P to the inscribed conic $\mathcal{C}_\ell$ with perspector the trilinear pole of ℓ_P. The two tangents for $i = 1, 2$ are perpendicular if and only if the line ℓ_P contains the orthocorrespondent $P^\perp$.*

For example, for $P = K$, the symmedian point, the circumconic $\mathcal{C}_P$ is the circumcircle. The orthocorrespondent is the point

$$K^\perp = (a^2(a^4 - b^4 + 4b^2c^2 - c^4) : \cdots : \cdots)$$

on the Euler line. The line ℓ joining K to this point has equation

$$\frac{(b^2-c^2)(b^2+c^2-2a^2)}{a^2}x + \frac{(c^2-a^2)(c^2+a^2-2b^2)}{b^2}y + \frac{(a^2-b^2)(a^2+b^2-2c^2)}{c^2}z = 0.$$

The inscribed conic $\mathcal{C}_\ell$ has center

$$(a^2(b^2-c^2)(a^4 - b^4 + b^2c^2 - c^4) : \cdots : \cdots).$$

The tangents from K to the conic $\mathcal{C}_\ell$ are the Brocard axis OK and its perpendicular at K. [10] The points of tangency are

$$\left(\frac{a^2(2a^2-b^2-c^2)}{b^2-c^2} : \frac{b^2(2b^2-c^2-a^2)}{c^2-a^2} : \frac{c^2(2c^2-a^2-b^2)}{a^2-b^2}\right)$$

on the Brocard axis and

$$\left(\frac{a^2(b^2-c^2)}{2a^2-b^2-c^2} : \frac{b^2(c^2-a^2)}{2b^2-c^2-a^2} : \frac{c^2(a^2-b^2)}{2c^2-a^2-b^2}\right)$$

on the perpendicular tangent. See Figure 7. The line ℓ intersects the circumcircle at the point

$$X_{110} = \left(\frac{a^2}{b^2-c^2} : \frac{b^2}{c^2-a^2} : \frac{c^2}{a^2-b^2}\right)$$

and the Parry point

$$X_{111} = \left(\frac{a^2}{b^2+c^2-2a^2} : \frac{b^2}{c^2+a^2-2b^2} : \frac{c^2}{a^2+b^2-2c^2}\right).$$

The lines containing the degenerate triangles of $\mathsf{Parasix}(K, X_{110})$ are parallel to the Brocard axis, while those for $\mathsf{Parasix}(K, X_{111})$ are parallel to the tangent from K which is perpendicular to the Brocard axis.

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[10] The infinite points of these lines are respectively X_{511} and X_{512}.

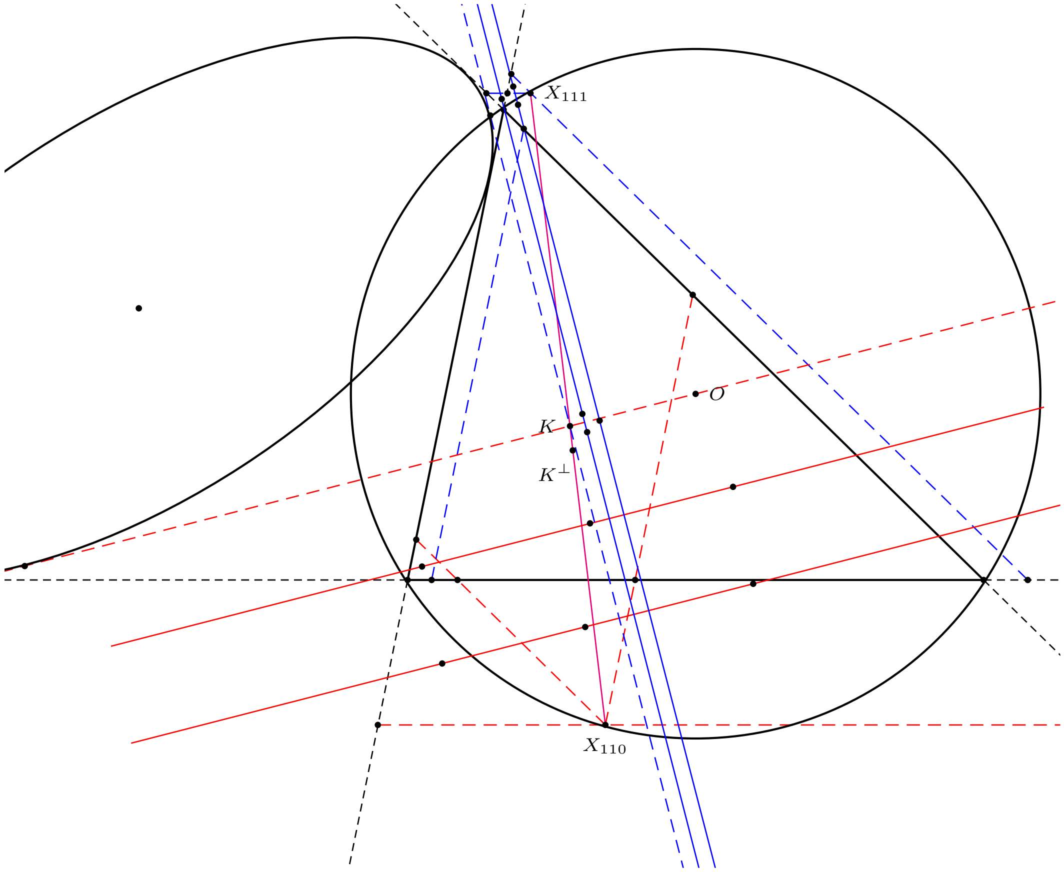

Figure 7. Degenerate $\mathsf{Parasix}(K, X_{110})$ and $\mathsf{Parasix}(K, X_{111})$

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