Forum Geometricorum
Volume 3 (2003) 205–206.

FORUM GEOM
ISSN 1534-1178

A 5-step Division of a Segment in the Golden Section

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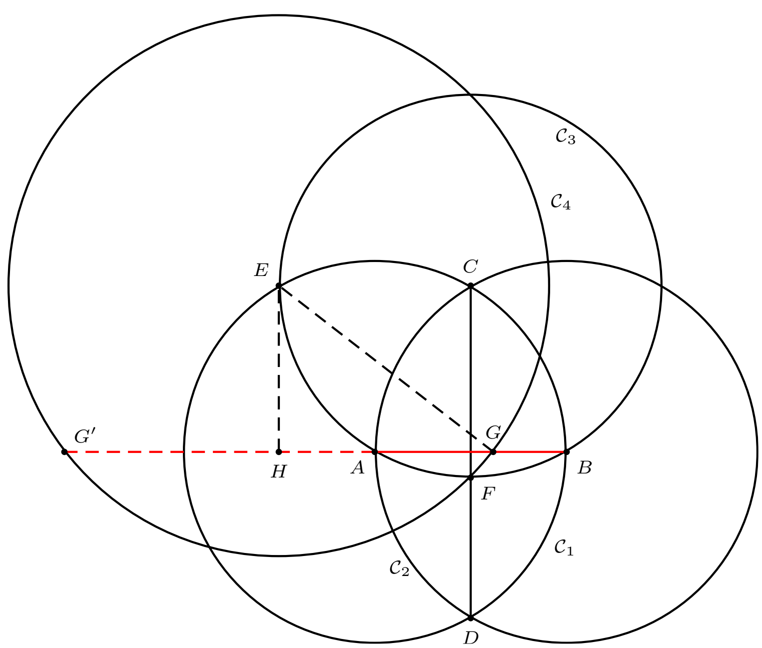
Abstract. Using ruler and compass only in five steps, we divide a given segment in the golden section.

Inasmuch as we have given in [1] a construction of the golden section by drawing 5 circular arcs, we present here a very simple division of a given segment in the golden section, in 5 euclidean steps, using ruler and compass only. For two points P and Q, we denote by $P(Q)$ the circle with P as center and PQ as radius.



Construction. Given a segment AB, construct

(1) $\mathcal{C}_1 = A(B)$,
(2) $\mathcal{C}_2 = B(A)$, intersecting $\mathcal{C}_1$ at C and D,
(3) $\mathcal{C}_3 = C(A)$, intersecting $\mathcal{C}_1$ again at E,
(4) the segment CD to intersect $\mathcal{C}_3$ at F,
(5) $\mathcal{C}_4 = E(F)$ to intersect AB at G.

The point G divides the segment AB in the golden section.

Publication Date: November 26, 2003. Communicating Editor: Paul Yiu.

Proof. Suppose AB has unit length. Then $CD = \sqrt{3}$ and $EG = EF = \sqrt{2}$. Let H be the orthogonal projection of E on the line AB. Since $HA = \frac{1}{2}$, and $HG^2 = EG^2 - EH^2 = 2 - \frac{3}{4} = \frac{5}{4}$, we have $AG = HG - HA = \frac{1}{2}(\sqrt{5} - 1)$. This shows that G divides AB in the golden section. □

Remark. The other intersection G' of $\mathcal{C}_4$ and the line AB is such that $G'A : AB = \frac{1}{2}(\sqrt{5} + 1) : 1$.

References

[1] K. Hofstetter, A simple construction of the golden section, *Forum Geom.*, 2 (2002) 65–66.

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