Some More Archimedean Circles in the Arbelos

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Abstract. We construct 4 circles in the arbelos which are congruent to the Archimedean twin circles.

Thomas Schoch [2] tells the remarkable story of his discovery in the 1970’s of the many Archimedean circles in the arbelos (shoemaker’s knife) that were eventually recorded in the paper [1]. In this note, we record four more Archimedean circles which were discovered in the summer of 1998, when the present author took a geometry course ([3]) with one of the authors of [1].

Consider an arbelos with inner semicircles \(C_1\) and \(C_2\) of radii \(a\) and \(b\), and outer semicircle \(C\) of radius \(a + b\). It is known the Archimedean circles have radius \(t = \frac{ab}{a+b}\). Let \(Q_1\) and \(Q_2\) be the “highest” points of \(C_1\) and \(C_2\) respectively.

Theorem. A circle tangent to \(C\) internally and to \(OQ_i\) at \(Q_i\) (or \(OQ_2\) at \(Q_2\)) has radius \(t = \frac{ab}{a+b}\).

Proof. There are two such circles tangent at \(Q_1\), namely, \((C'_1)\) and \((C''_1)\) in Figure 1. Consider one such circle \((C_1)\) with radius \(r\). Note that

\[ OQ_1^2 = O_1Q_1^2 + OO_1^2 = a^2 + b^2. \]

It follows that

\[ (a + b - r)^2 = (a^2 + b^2) + r^2, \]

from which \(r = \frac{ab}{a+b} = t\). The same calculation shows that \((C'_1)\) also has radius \(t\), and similarly for the two circles at \(Q_2\).
References


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