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Some More Archimedean Circles in the Arbelos

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Abstract. We construct 4 circles in the arbelos which are congruent to the Archimedean twin circles.

Thomas Schoch [2] tells the remarkable story of his discovery in the 1970's of the many Archimedean circles in the arbelos (shoemaker's knife) that were eventually recorded in the paper [1]. In this note, we record four more Archimedean circles which were discovered in the summer of 1998, when the present author took a geometry course ([3]) with one of the authors of [1].

Consider an arbelos with inner semicircles $\mathcal{C}_1$ and $\mathcal{C}_2$ of radii a and b, and outer semicircle $\mathcal{C}$ of radius $a+b$. It is known the Archimedean circles have radius $t = \frac{ab}{a+b}$. Let Q_1 and Q_2 be the "highest" points of $\mathcal{C}_1$ and $\mathcal{C}_2$ respectively.

Theorem. *A circle tangent to $\mathcal{C}$ internally and to OQ_1 at Q_1 (or OQ_2 at Q_2) has radius $t = \frac{ab}{a+b}$.*

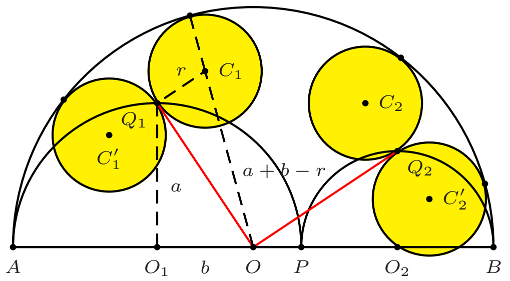

Figure 1

Proof. There are two such circles tangent at Q_1, namely, (C_1) and (C_1') in Figure 1. Consider one such circle (C_1) with radius r. Note that

$$OQ_1^2 = O_1Q_1^2 + OO_1^2 = a^2 + b^2.$$

It follows that

$$(a+b-r)^2 = (a^2+b^2) + r^2,$$

from which $r = \frac{ab}{a+b} = t$. The same calculation shows that (C_1') also has radius t, and similarly for the two circles at Q_2. □

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References

[1] C. W. Dodge, T. Schoch, P. Y. Woo and P. Yiu, Those ubiquitous Archimedean circles, *Math. Mag.*, 72 (1999) 202–213.
[2] T. Schoch, Arbelos, http://www.retas.de/thomas/arbelos/arbelos.html.
[3] P. Yiu, *Euclidean Geometry*, available at http://www.math.fau.edu/Yiu/Geometry.html.

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