# Divison of a Segment in the Golden Section with Ruler and Rusty Compass 

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#### Abstract

We give a simple 5-step division of a segment into golden section, using ruler and rusty compass.


In [1] we have given a 5 -step division of a segment in the golden section with ruler and compass. We modify the construction by using a rusty compass, i.e., one when set at a particular opening, is not permitted to change. For a point $P$ and a segment $A B$, we denote by $P(A B)$ the circle with $P$ as center and $A B$ as radius.


Figure 1

Construction. Given a segment $A B$, construct
(1) $\mathcal{C}_{1}=A(A B)$,
(2) $\mathcal{C}_{2}=B(A B)$, intersecting $\mathcal{C}_{1}$ at $C$ and $D$,
(3) the line $C D$ to intersect $A B$ at its midpoint $M$,
(4) $\mathcal{C}_{3}=M(A B)$ to intersect $\mathcal{C}_{2}$ at $F$ (so that $C$ and $D$ are on opposite sides of $A B$ ),
(5) the segment $C F$ to intersect $A B$ at $G$.

The point $G$ divides the segment $A B$ in the golden section.


Figure 2
Proof. Extend $B A$ to intersect $\mathcal{C}_{1}$ at $E$. According to [1], it is enough to show that $E F=2 \cdot A B$. Let $F^{\prime}$ be the orthogonal projection of $F$ on $A B$. It is the midpoint of $M B$. Without loss of generality, assume $A B=4$, so that $M F^{\prime}=F^{\prime} B=1$ and $E F^{\prime}=2 \cdot A B-F^{\prime} B=7$. Applying the Pythagorean theorem to the right triangles $E F F^{\prime}$ and $M F F^{\prime}$, we have

$$
\begin{aligned}
E F^{2} & =E F^{\prime 2}+F F^{\prime 2} \\
& =E F^{\prime 2}+M F^{2}-M F^{2} \\
& =7^{2}+4^{2}-1^{2} \\
& =64 .
\end{aligned}
$$

This shows that $E F=8=2 \cdot A B$.

## References

[1] K. Hofstetter, Another 5-step division of a segment in the golden section, Forum Geom., 4 (2004) 21-22.

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