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A 4-Step Construction of the Golden Ratio

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Abstract. We construct, in 4 steps using ruler and compass, three points two of the distances between which bear the golden ratio.

We present here a 4-step construction of the golden ratio using ruler and compass only. More precisely, we construct, in 4 steps using ruler and compass, three points with two distances bearing the golden ratio. It is fascinating to discover how simple the golden ratio appears. We denote by $P(Q)$ the circle with center P, passing through Q, and by $P(XY)$ that with center P and radius XY.

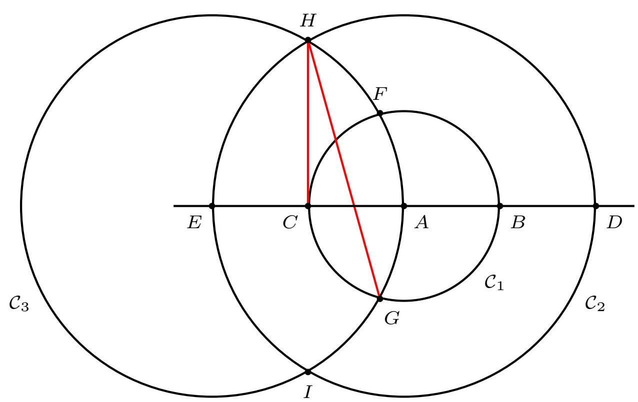

Figure 1

Construction. Given two points A and B, construct

(1) the circle $\mathcal{C}_1 = A(B)$,
(2) the line AB to intersect $\mathcal{C}_1$ again at C and extend it long enough to intersect
(3) the circle $\mathcal{C}_2 = A(BC)$ at D and E,
(4) the circle $\mathcal{C}_3 = E(BC)$ to intersect $\mathcal{C}_1$ at F and G, and $\mathcal{C}_2$ at H and I.

Then $\frac{GH}{CH} = \frac{\sqrt{5}+1}{2}$.

Proof. Without loss of generality let $AB = 1$, so that $BC = AE = AH = EH = 2$. Triangle AEH is equilateral. Let $\mathcal{C}_4 = H(A)$, intersecting $\mathcal{C}_1$ at J. By symmetry, AGJ is an equilateral triangle. Let $\mathcal{C}_5 = J(A) = J(AG) = J(AB)$, intersecting $\mathcal{C}_1$ at K. Finally, let $\mathcal{C}_6 = J(H) = J(BC)$. See Figure 2.

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With $\mathcal{C}_1$, $\mathcal{C}_5$, $\mathcal{C}_2$, $\mathcal{C}_6$, following [1], K divides GH in the golden section. It suffices to prove $CH = GK = \sqrt{3}$. This is clear for GK since the equilateral triangles AJG and AJK have sides of length 1. On the other hand, in the right triangle ACH, $AC = 1$ and $AH = 2$. By the Pythogorean theorem $CH = \sqrt{3}$. □

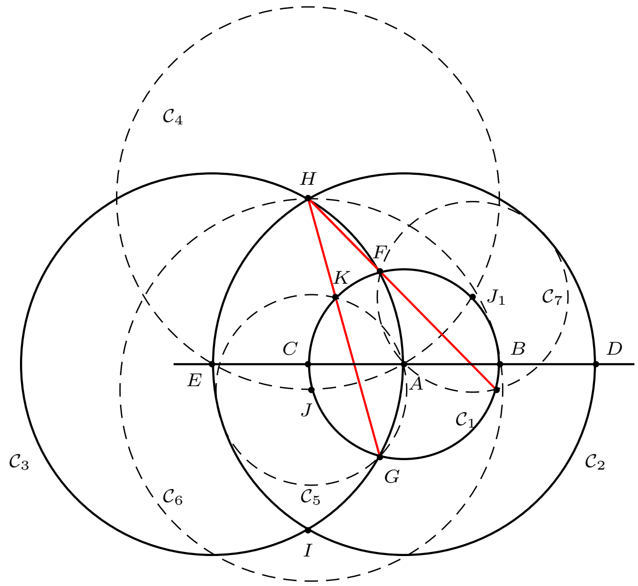

Figure 2

Remark. $\frac{CH}{FH} = \frac{GH}{CH} = \frac{\sqrt{5}+1}{2}$.

Proof. Since $CH^2 = GH \cdot KH$, it is enough to prove that $FH = KH$. Let $\mathcal{C}_4$ intersect $\mathcal{C}_1$ again at J_1. Consider the circle $\mathcal{C}_7 = J_1(A)$. By symmetry, F lies on $\mathcal{C}_7$ and $FH = KH$. □

Reference

[1] K. Hofstetter, A simple construction of the golden section, *Forum Geom.*, 2 (2002) 65–66.

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