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A Simple Construction of the Golden Ratio

Jingcheng Tong and Sidney Kung

Abstract. We construct the golden ratio by using an area bisector of a trapezoid.

Consider a trapezoid $PQRS$ with bases $PQ = b$, $RS = a$, $a < b$. Assume, in Figure 1, that the segment MN of length $\sqrt{\frac{a^2+b^2}{2}}$ is parallel to PQ. Then MN lies between the bases PQ and RS (see [1, p.57]). It is easy to show that MN bisects the area of the trapezoid. It is more interesting to note that M and N divide SP and RQ in the golden ratio if $b = 3a$. To see this, construct a segment SW parallel to RQ and let $V = MN \cap SW$. It is clear that

$$\frac{SM}{SP} = \frac{MV}{PW} = \frac{\sqrt{\frac{a^2+b^2}{2}} - a}{b-a} = \frac{\sqrt{5}-1}{2}$$

if $b = 3a$.

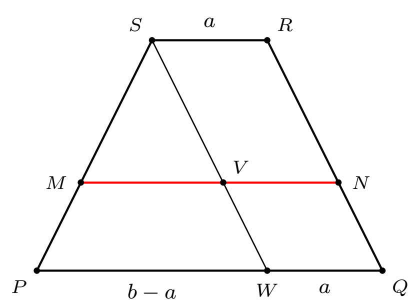

Figure 1

Based upon this result, we present the following simple division of a given segment AB in the golden ratio. Construct
(1) a trapezoid $ABCD$ with $AD//BC$ and $BC = 3 \cdot AD$,
(2) a right triangle BCD with a right angle at C and $CE = AD$,
(3) the midpoint F of BE and a point H on the perpendicular bisector of BE such that $FH = \frac{1}{2}BE$,
(4) a point I on BC such that $BI = BH$.

Complete a parallelogram $BIJG$ with J on DC and G on AB. See Figure 2. Then G divides AB in the golden ratio, *i.e.*, $\frac{AB}{AB} = \frac{\sqrt{5}-1}{2}$.

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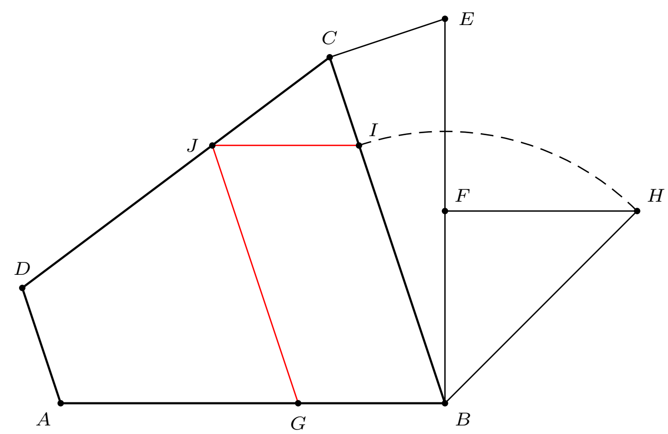

Figure 2

Proof. The trapezoid $ABCD$ has $AD = a$, $BC = b$ with $b = 3a$. The segment JG is parallel to the bases and

$$JG = BI = BH = \sqrt{2} \cdot \frac{\sqrt{a^2 + b^2}}{2} = \sqrt{\frac{a^2 + b^2}{2}}.$$

Therefore, $\frac{AG}{AB} = \frac{\sqrt{5}-1}{2}$. □

Reference

[1] R. B. Nelsen, *Proofs Without Words*, Mathematical Association of America, 1993.

Jingcheng Tong: Department of Mathematics, University of North Florida, Jacksonville, Florida, 32224, USA
E-mail address: `jtong@unf.edu`

Sidney Kung: 20488 Stevens Creek Blvd., #1411, Cupertino, California, 95014, USA
E-mail address: `sidneykung@yahoo.com`