

A Simple Compass-Only Construction of the Regular Pentagon

Kurt Hofstetter

Abstract. In 7 steps we give a simple compass-only (Mascheroni) construction of the vertices of a regular pentagon .

In [1] we have given a simple 5-step compass-only (Mascheroni) construction of the golden section. Here we note that with two additional circles, it is possible to construct the vertices of a regular pentagon. As usual, we denote by $P(Q)$ the circle with center P and passing through Q .

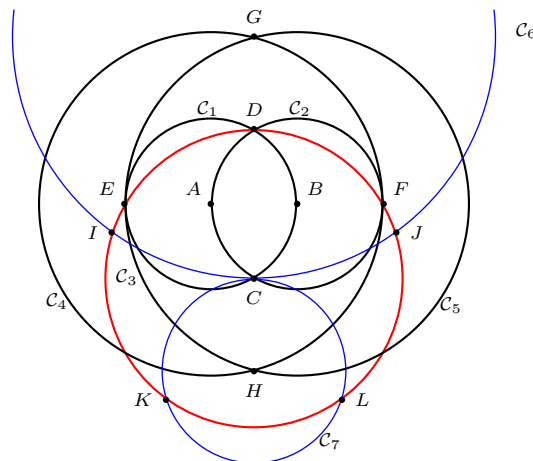


Figure 1

Construction 1. Given two points A and B ,

- (1) $C_1 = A(B)$,
- (2) $C_2 = B(A)$ to intersect C_1 at C and D ,
- (3) $C_3 = C(D)$ to intersect C_1 at E and C_2 at F ,
- (4) $C_4 = A(F)$,
- (5) $C_5 = B(E)$ to intersect C_4 at G and H .
- (6) $C_6 = G(C)$ to intersect C_3 at I and J ,
- (7) $C_7 = H(C)$ to intersect C_3 at K and L .

Then $DIK LJ$ is a regular pentagon.

Proof. In [1] we have shown that the first five steps above lead to four collinear points C, D, G, H such that D divides CG , and C divides DH , in the golden section.

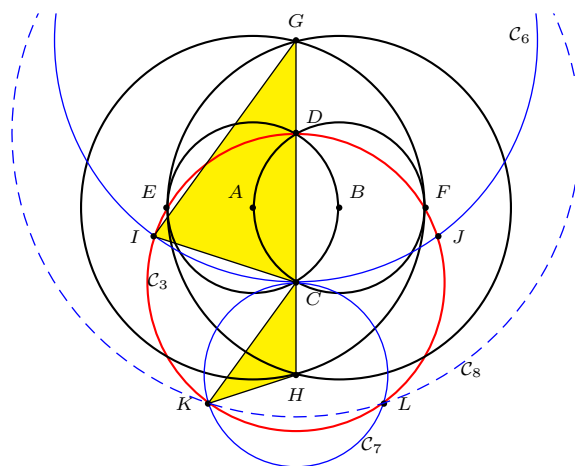


Figure 2

(i) This means that in the isosceles triangle GCI , $\frac{GC}{IC} = \frac{GC}{DC} = \phi$. The base angles are 72° . Therefore, $\angle DCI = 72^\circ$. By symmetry, $\angle DCJ = 72^\circ$.

(ii) Also, in the isosceles triangle HCK , $\frac{KC}{CH} = \frac{DC}{CH} = \phi$. The base angles are 36° . It follows that $\angle KCH = 36^\circ$. By symmetry, $\angle LCH = 36^\circ$, and $\angle KCL = 72^\circ$.

(iii) Since C is on the line GH , $\angle ICK = 180^\circ - \angle GCI - \angle KCH = 72^\circ$. By symmetry, $\angle JCL = 72^\circ$.

Therefore, the five points D, I, K, L, J are equally spaced on the circle C_3 . They form the vertices of a regular pentagon. \square

Remark. The circle C_7 can be replaced by C_8 with center D and radius IJ . This intersects C_3 at the same points K and L .

Reference

[1] K. Hofstetter, A simple construction of the golden section, *Forum Geom.*, 2 (2002) 65–66.

Kurt Hofstetter: Object Hofstetter, Media Art Studio, Langegasse 42/8c, A-1080 Vienna, Austria
E-mail address: hofstetter@sunpendulum.at