

Two More Powerian Pairs in the Arbelos

Quang Tuan Bui

Abstract. We construct two more pairs of Archimedes circles analogous to those of Frank Power, in addition to those by Floor van Lamoen and the author.

Consider an arbelos with semicircles (O) , (O_1) , (O_2) with diameters AB , AC , BC as diameters respectively. Denote by r_1 and r_2 respectively the radii of (O_1) and (O_2) , and D the intersection of (AB) with the perpendicular to AB at C . If P is a point such that $OP^2 = r_1^2 + r_2^2$, then the circles tangent to (O) and to OP at P are Archimedean. Examples were first given in Power [3], subsequently also in [1, 2].

We construct two more Powerian pairs.

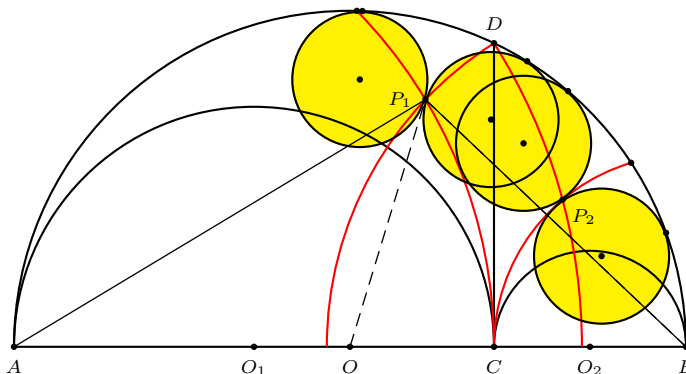


Figure 1

Let P_1 be the intersection of the circles $A(C)$ and $B(D)$. Consider OP_1 as a median of triangle P_1AB , we have, by Apollonius' theorem (see, for example, [4]),

$$\begin{aligned} OP_1^2 &= \frac{1}{2} (AP_1^2 + BP_1^2) - OA^2 \\ &= \frac{1}{2} ((2r_1)^2 + 2r_2 \cdot 2(r_1 + r_2)) - (r_1 + r_2)^2 \\ &= r_1^2 + r_2^2. \end{aligned}$$

Similarly, for P_2 the intersection of $B(C)$ and $A(D)$, $OP_2^2 = r_1^2 + r_2^2$. Therefore, we have two Powerian pairs at P_1, P_2 .

References

- [1] Q. T. Bui, The arbelos and nine-point circles, *Forum Geom.*, 7 (2007) 115–120.
- [2] F. M. van Lamoen, Some more Powerian pairs in the arbelos, *Forum Geom.*, 7 (2007) 111–113.
- [3] F. Power, Some more Archimedean circles in the arbelos, *Forum Geom.*, 5 (2005) 133–134.
- [4] P. Yiu, *Euclidean Geometry*, Florida Atlantic University Lecture Notes, 1998,
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Quang Tuan Bui: 45B, 296/86 by-street, Minh Khai Street, Hanoi, Vietnam
E-mail address: bqtuan1962@yahoo.com