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Golden Sections in a Regular Hexagon

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Abstract. We relate a golden section associated with a regular hexagon to two recent simple constructions by Hofstetter and Bataille, and give a large number of golden sections of segments in a regular hexagon.

Consider a regular hexagon $ABCDEF$ with center O. Let M be a point on the side BC. An equilateral triangle constructed on AM has its third vertex N on the radius OE, such that $ON = BM$.

Proposition 1. *The area of the regular hexagon $ABCDEF$ is 3 times the area of triangle AMN if and only if M divides BC in the golden ratio.*

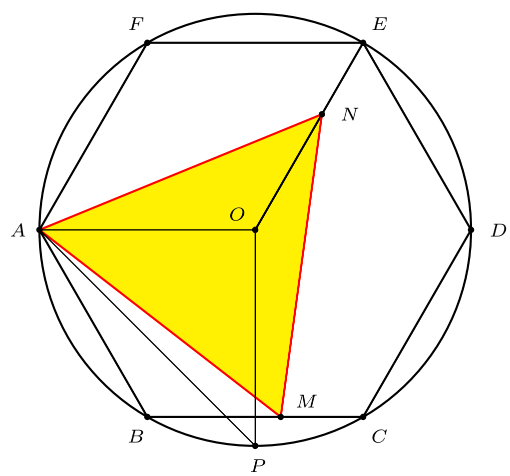

Figure 1.

Proof. Let P is midpoint of the minor BC. It is clear that AOP is an isosceles right triangle and $AP = \sqrt{2} \cdot AO$ (see Figure 1). The area of the regular hexagon $ABCDEF$ is three times that of triangle AMN if and only if

$$\frac{\Delta AMN}{\Delta ABO} = 2 \Leftrightarrow \frac{AM}{AO} = \sqrt{2} \Leftrightarrow AM = AP.$$

Equivalently, M is an intersection of BC with the circle $O(P)$. We fill in the circles $B(C)$, $C(B)$ and $A(P)$. These three circles execute exactly Hofstetter's division of the segment BC in the golden ratio at the point M [2] (see Figure 2).[1] □

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[1]Hofstetter has subsequently noted [3] that this construction was known to E. Lemoine and J. Reusch one century ago.

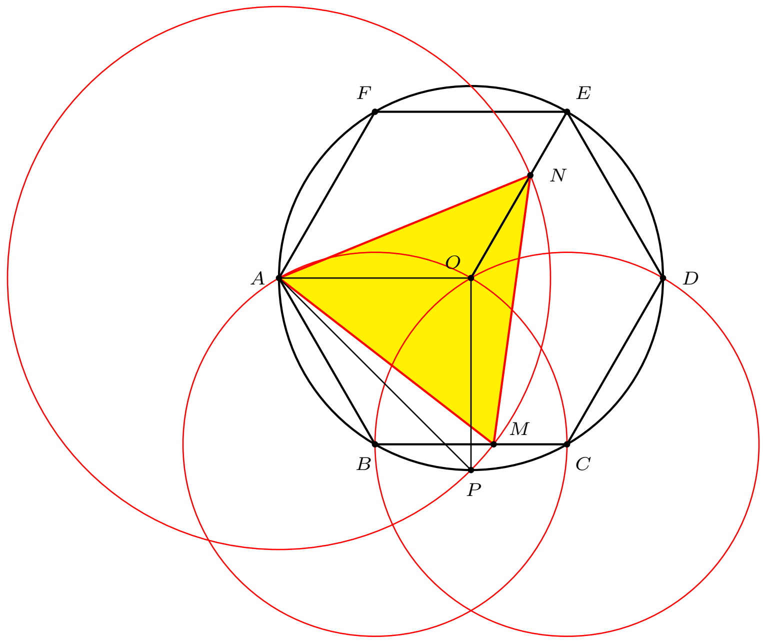

Figure 2.

Since triangles AON and ABM are congruent, N also divides OE in the golden ratio. This fact also follows independently from a construction given by M. Bataille. Let Q be the antipodal point of P, and complete the square $AOQR$. According to [1], O divides BN in the golden ratio. Since O is the midpoint of BE, it follows easily that N divides OE in the golden ratio as well.

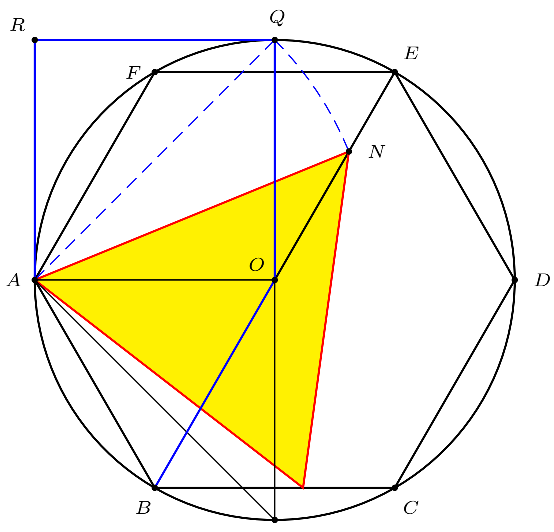

Figure 3.

Proposition 2. *The sides of the equilateral triangle AMN are divided in the golden ratio as follows.*

Directed segment	MN	NM	AM	AN	NA	MA
divided by	OD	OC	OB	OF	BF	perp. from O to AB
in golden ratio at	A_n	A_m	N_m	M_n	M_a	N_a

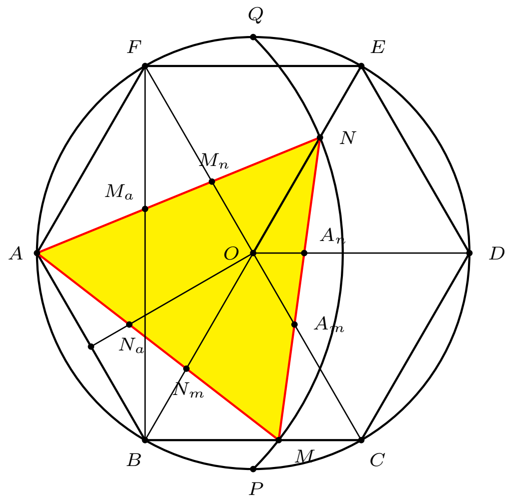

Figure 4

Proposition 3. *Each of the six points A_m, A_n, M_n, M_a, N_a, N_m divides a segment, apart from the sides of the equilateral triangle AMN, in the golden ratio.*

Point	A_m	A_n	M_n	M_a	N_a	N_m
Segment	CO	AD	FO	BF	SR	OB

Here, S is the midpoint of ON_a.

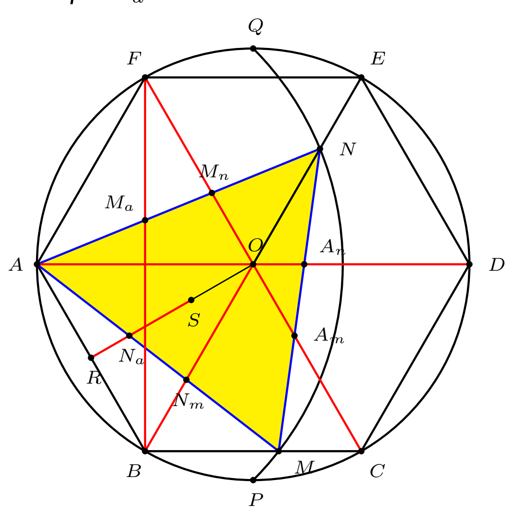

Figure 5.

Proposition 4. *The segments N_aM_a, N_mA_m and M_nA_n are divided in the golden ratio by the lines OA, OP, OE respectively.*

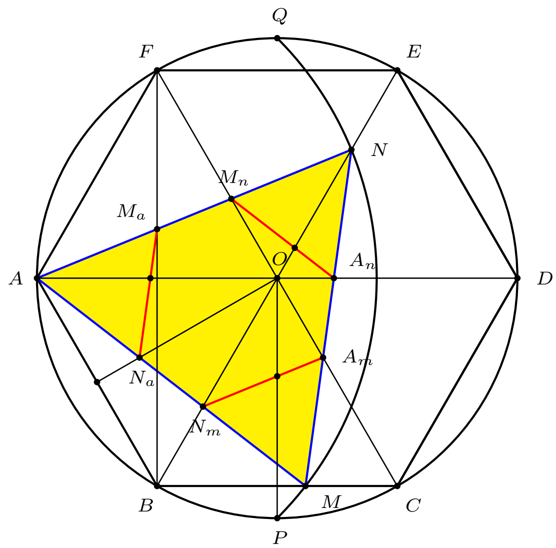

Figure 6.

References

[1] M. Bataille, Another simple construction of the golden section, *Forum Geom.*, 11 (2011) 55.

[2] K. Hofstetter, A 5-step division of a segment in the golden section, *Forum Geom.*, 3 (2003) 205–206.

[3] K. Hofstetter, Another 5-step division of a segment in the golden section, *Forum Geom.*, 4 (2004) 21–22.

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