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A Special Point in the Arbelos Leading to a Pair of Archimedean Circles

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Abstract. In 2011 Quang Tuan Bui found a beautiful and simple pair of Archimedean circles, which were published on a website. From this pair we find a special point in the Arbelos leading to a related pair of Archimedean circles.

In 2011 Quang Tuan Bui found a beautiful and elegant pair of Archimedean circles. These were published by Alexander Bogomolny on his website [1]. In 2013 the circle pair was found independently by Hiroshi Okumura [2, 3].

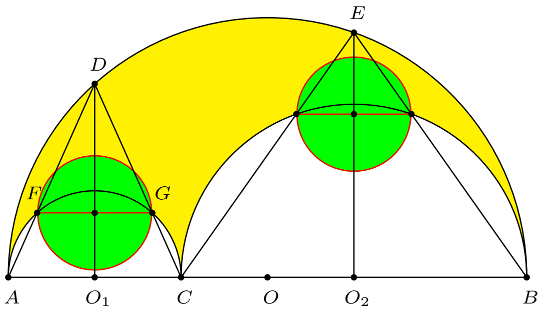

Figure 1.

Consider an arbelos with (O) being the semicircle with diameter AB, while the point C on AB defines the smaller semicircles (O_1) and (O_2) on AC and BC respectively. Let the perpendiculars to AB from O_1 and O_2 meet (O) in D and E respectively. The segments DA and DC meet (O_1) in two points F and G. The segment FG is the diameter of an Archimedean circle (see Figure 1). Likewise an Archimedean circle is found from E.

To prove the correctness of the finding of Bui, we let r, r_1 and r_2 be the radii of (O), (O_1) and (O_2) respectively. Note that $AF : AD = r_1 : r$, so that $AD : FD = r : r_2$. Of course G divides CD in the same ratio. So, by similarity $FG = \frac{r_2}{r} \cdot AC = 2 \cdot \frac{r_1 r_2}{r}$. the Archimedean diameter.

Now one may wonder what the locus of points P is such that PA and PC cut a chord ST off (O_1) congruent to FG. See Figure 2.

For ST to be congruent to FG, it is clear that arcs FS and GT must be congruent. From this the angles DAP and DCP must be congruent, and we conclude that $ACDP$ is cyclic. The locus of P is thus the circumcircle of triangle ACD.

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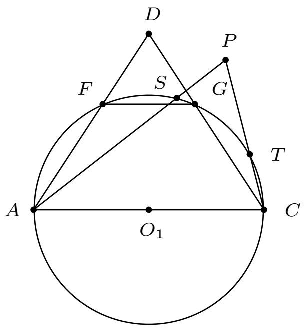

Figure 2.

Similarly, the locus of P for PB and PC to cut congruent to the one cut out by EB and EC is the circumcircle BCE. Now the circumcircles of ACD and BCE intersect, apart from C, in a point L. This point is thus the only point leading to an Archimedean circle on each of the semicircles (O_1) and (O_2). A notable point (see Figure 3).

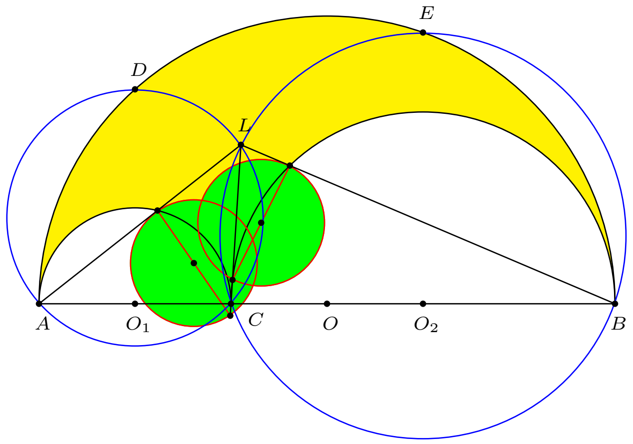

Figure 3.

References

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