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The Golden Section in the Inscribed Square of an Isosceles Right Triangle

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Abstract. We prove the occurrence of the golden section with the inscribed square of an isosceles right triangle on its hypotenuse and its circumcircle.

Given a right isosceles triangle ABC and its circumcircle, inscribed a square $DEFG$ with a side FG along the hypotenuse AB. If the side DE is extended to intersect the circumcircle at P, then E divides DP in the golden ratio (see Figure 1). This is reminiscent of the golden section by Odom's construction [2]; see also [1].

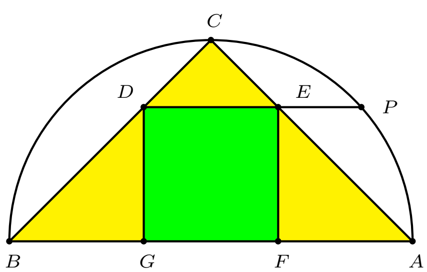

Figure 1

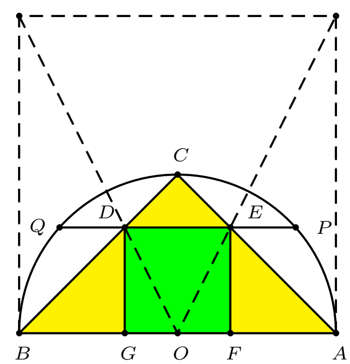

Figure 2

A simple construction of the inscribed square (see Figure 2) leads to a simple calculation giving the ratio $\frac{DP}{DE} = \frac{\sqrt{5}+1}{2}$, the golden ratio. We give a synthetic proof here.

From the similarity of the isosceles right triangles DEC and AEF, we have

$$\frac{DE}{EC} = \frac{AE}{EF} \implies DE^2 = DE \cdot EF = AE \cdot EC.$$

If the line DE intersects the circumcircle again at Q, then $EQ = DP$. By the intersecting chords theorem, $AE \cdot EC = PE \cdot EQ = PE \cdot DP$. Therefore, $DE^2 = EP \cdot DP$, and E divides DP in the golden ratio.

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References

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[2] G. Odom and J. van de Craats, Elementary Problem 3007, *Amer. Math. Monthly*, 90 (1983) 482; solution, 93 (1986) 572.

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