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Two Pairs of Archimedean Circles Derived from a Square

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Abstract. We construct two pairs of Archimedean circles in the square built on the base and on the same side of an arbelos.

Consider an arbelos with two inner semicircles α, β with diameters AO, BO and radii a and b, respectively for a point O on the segment AB. The perpendicular to AB at O is called the axis. It is well known that the two Archimedean circles each tangent to the axis, the outer semicircle (with diameter AB), and to α, β respectively have a common radius $R_{\rm A} = \frac{ab}{a+b}$ (see [1] and Figure 1).

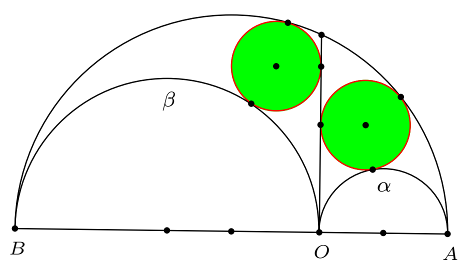

Figure 1

Construct a square $ABDC$ on the same side of AB as the arbelos. Let t_α be the tangent of the semicircle α parallel to AB with point of tangency T_α. Similarly the line t_β and the point T_β are defined. Let E be the intersection of the lines CO and t_β. Since the triangle formed by CO, CD and the axis and the triangle formed by CO, t_β and the axis are similar, the distance from E to the axis equals $\frac{2a}{2a+2b} \cdot b = \frac{ab}{a+b} = R_{\rm A}$. Hence the circle with center E touching the axis is Archimedean (see Figure 2).

Let F be the the point of intersection of CO and t_α. The distance between F and the axis equals $\frac{2a}{2a+2b} \cdot a = \frac{a^2}{a+b} = a - \frac{ab}{a+b} = a - R_{\rm A}$. Hence the circle with center F and passing through the point T_α is also Archimedean.

Similarly, there are two Archimedean circles with centers at the intersections of DO and the tangents t_α, t_β. Thus, we have constructed two pairs of Archimedean circles. The centers of these circles, and their orthogonal projections on AB form the vertices of two squares with side lengths a and b respectively.

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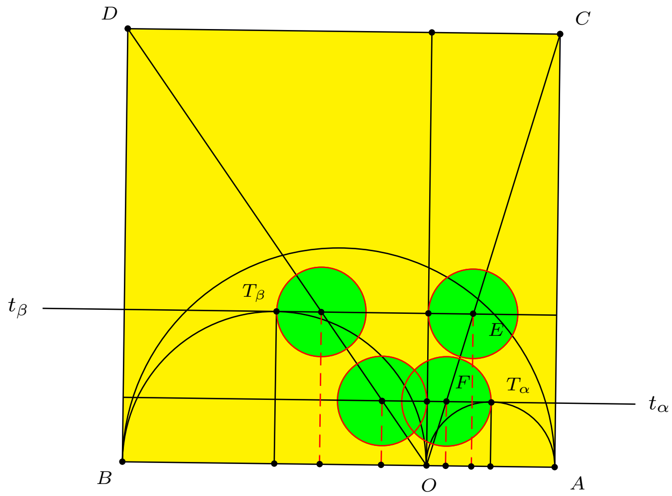

Figure 2

Reference

[1] C. W. Dodge, T. Schoch, P. Y. Woo and P. Yiu, Those ubiquitous Archimedean circles, *Math. Mag.*, 72 (1999) 202–213.

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