Forum Geometricorum
Volume 17 (2017) 45–46.

FORUM GEOM
ISSN 1534-1178

Do Dogs Know The Bifurcation Locus?

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Abstract. A dog runs at speed r and swims at speed s, with $r > s$. For a fixed point A in a lake with a straight shoreline, where are all the points B in the lake such that the direct swimming path from A to B takes the dog the same time as the fastest indirect swimming- running-swimming path? We give a simple geometric solution to this bifurcation-locus problem.

In [1], the authors discuss a challenging situation for the remarkable dog Elvis: As in Figure 1, Elvis is initially at a point A in the lake and a ball is thrown to a point B in the lake. What path should Elvis choose in order to minimize his time to reach the ball? This is challenging because Elvis runs at speed r and swims at speed s, with $r > s$, so a direct swimming (S) path AB may be slower than an indirect swimming-running-swimming (SRS) path $AXYB$.

Figure 1. Elvis' dilemma

In [2], the author gives a simple ruler-compass determination of the optimal path (S or the fastest SRS) for any two given points A and B.

We now ask a more interesting question. For a fixed point A, a point B is called a bifurcation point of A if the S-path from A to B takes the same time as the fastest SRS-path from A to B. What is the locus of bifurcation points of A? This question has a nice answer and a simple geometric proof.

As in Figure 2, let B be a bifurcation point of A. The fastest SRS-path from A to B is $AXY\ B$ where AX and YB form the angle $\theta = \arccos \frac{s}{r}$ with the shoreline (see [1] or [2]). Let A' be the reflection of A across the shoreline. Draw the line d through A' and perpendicular to $A'X$.

Publication Date: February 24, 2017. Communicating Editor: Paul Yiu.

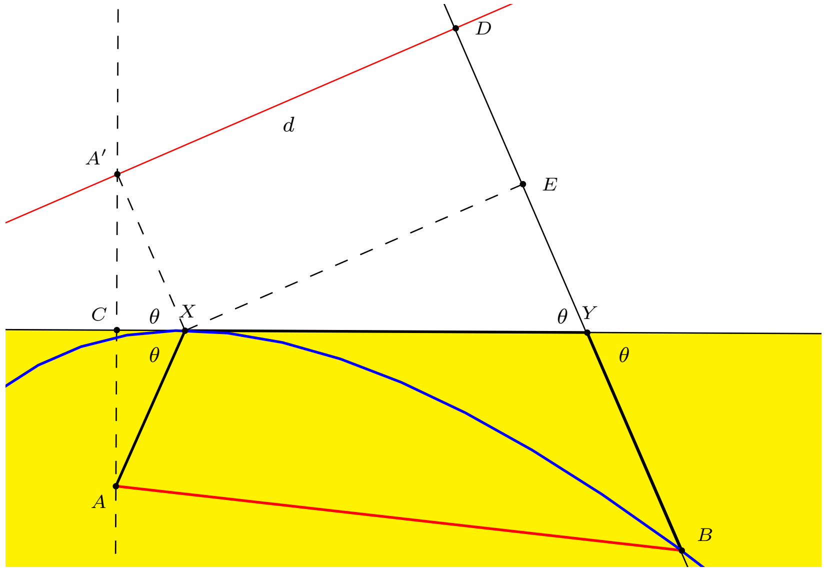

Figure 2. Bifurcation locus of A

Theorem 1. *The bifurcation locus of A is part of the parabola with focus A and directrix d.*

Proof. Note that $BY \perp d$ with foot D. Locate E on BD such that $XE \parallel d$. Then $AX = A'X = DE$, and the dog swims the distance EY in the same time as he runs the distance XY. Thus, the time to swim the distance BD is the same as the time for the SRS-path $AXYB$, thus also the same as the time to swim the distance AB. Hence, $BD = BA$, completing the proof.

Of course, the locus is the part of the parabola starting at X and moving away from A. □

References

[1] R. Minton and T. J. Pennings, Do dogs know bifurcations?, *College Math. J.*, 38 (2007) 356–361.
[2] L. Zhou, Do dogs play with rulers and compasses?, *Forum Geom.*, 15 (2015) 159–164.

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