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Orthopoles, Flanks, and Vecten Points

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Abstract. Erect a square outwardly (or inwardly) from each side of the triangle. For the flank lines construct orthopoles. The lines from this orthopoles to corresponding vertexes of triangle concur in the Vecten point (or in the Inner Vecten point). This fact is showing together with a number of other interesting results, which include complex coordinates of Vecten points and orthopole.

1. Introduction

We will use complex numbers in the proofs.

Denote by a lowercase letter the complex coordinate of a point denoted by an uppercase letter.

Let ABC be a triangle. With no loss of generality, we can say that the circumcircle $\mathcal{C}$ is the union circle. Then $a.\bar{a} = b.\bar{b} = c.\bar{c} = 1$.

1.1. *The Outer Vecten point.*

Theorem 1. *Erect a square outwardly from each side of triangle ABC. Let $A_oB_oC_o$ be the triangle formed by the respective centers of the squares. The lines AA_o, BB_o, CC_o concur in X_{485}.* [1]

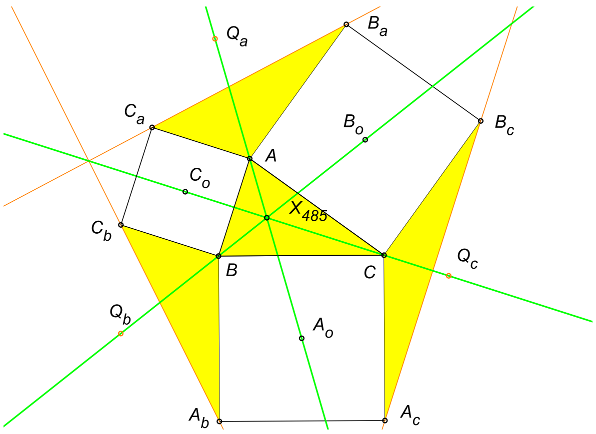

Figure 1. The Outer Vecten point

Point X_{485} is the Outer Vecten point.

Proof. Let the squares are: AC_aC_bB, BA_bA_cC, and CB_cB_aA with centers C_o, A_o, B_o respectively (Figure 1).

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Then $a_b = b + (b-c)i$, $b_c = c + (c-a)i$, $c_a = a + (a-b)i$, $\bar{a}_b = \frac{c+(b-c)i}{bc}$, $\bar{b}_c = \frac{a+(c-a)i}{ca}$, $\bar{c}_a = \frac{b+(a-b)i}{ab}$.

The coordinates of A_o, B_o, C_o and its complex conjugates are

$$\begin{aligned} a_o &= \frac{1}{2}(a_b + c) = \frac{1}{2}((b+c) + (b-c)i) \\ \bar{a}_o &= \frac{(b+c)+(b-c)i}{2bc} \\ b_o &= \frac{1}{2}(b_c + a) = \frac{1}{2}((c+a) + (c-a)i) \\ \bar{b}_o &= \frac{(c+a)+(c-a)i}{2ca} \\ c_o &= \frac{1}{2}(c_a + b) = \frac{1}{2}((a+b) + (a-b)i) \\ \bar{c}_o &= \frac{(a+b)+(a-b)i}{2ab} \end{aligned} \tag{1}$$

The line AA_o, joining a and a_o has equation

$$\begin{aligned} 0 &= \begin{vmatrix} z & a & a_o \\ \bar{z} & \bar{a} & \bar{a}_o \\ 1 & 1 & 1 \end{vmatrix} = (\bar{a}_o - \bar{a})z - (a_o - a)\bar{z} + (a_o\bar{a} - \bar{a}_o a) \\ &= \frac{\bar{a}_o - \bar{a}}{a_o - a} z - \bar{z} + \frac{a_o\bar{a} - \bar{a}_o a}{a_o - a} \end{aligned} \tag{2}$$

Lines BB_o and CC_o have equations

$$\begin{aligned} 0 &= (\bar{b}_o - \bar{b})z - (b_o - b)\bar{z} + (b_o\bar{b} - \bar{b}_o b) \\ &= \frac{\bar{b}_o - \bar{b}}{b_o - b} \cdot z - \bar{z} + \frac{b_o\bar{b} - \bar{b}_o b}{b_o - b} \end{aligned} \tag{3}$$

$$\begin{aligned} 0 &= (\bar{c}_o - \bar{c})z - (c_o - c)\bar{z} + (c_o\bar{c} - \bar{c}_o c) \\ &= \frac{\bar{c}_o - \bar{c}}{c_o - c} \cdot z - \bar{z} + \frac{c_o\bar{c} - \bar{c}_o c}{c_o - c} \end{aligned} \tag{4}$$

The intersection of the last two lines is given by

$$\begin{aligned} 0 &= \left(\frac{\bar{b}_o - \bar{b}}{b_o - b} - \frac{\bar{c}_o - \bar{c}}{c_o - c}\right) \cdot z + \left(\frac{b_o\bar{b} - \bar{b}_o b}{b_o - b} - \frac{c_o\bar{c} - \bar{c}_o c}{c_o - c}\right) \\ &= \frac{-2(a-b)(b-c)(c-a) - i(a(b-c)^2 + b(c-a)^2 + c(a-b)^2)}{2abc} \cdot z \\ &+ \frac{(a-b)(b-c)(c-a)(a+b+c) + i(a^2(b-c)^2 + b^2(c-a)^2 + c^2(a-b)^2)}{2abc} \\ z &= \frac{(a-b)(b-c)(c-a)(a+b+c) + i(a^2(b-c)^2 + b^2(c-a)^2 + c^2(a-b)^2)}{2(a-b)(b-c)(c-a) + i(a(b-c)^2 + b(c-a)^2 + c(a-b)^2)} \end{aligned}$$

Note that this is symmetric in a, b, c. This means that the three lines AA_o, BB_o, CC_o are concurrent. The point of concurrency is the Vecten point X_{485}. □

Corollary 2. *Let the circumcircle of triangle ABC be the unit circle. The Outer Vecten point X_{485} has coordinates*

$$x_{485} = \frac{(a-b)(b-c)(c-a)(a+b+c) + i(a^2(b-c)^2 + b^2(c-a)^2 + c^2(a-b)^2)}{2(a-b)(b-c)(c-a) + i(a(b-c)^2 + b(c-a)^2 + c(a-b)^2)} \tag{5}$$

1.2. *The Inner Vecten point.*

Theorem 3. *Erect a square inwardly from each side of triangle ABC. Let $A'_oB'_oC'_o$ be the triangle formed by the respective centers of the squares. The lines AA'_o, BB'_o, CC'_o concur in X_{486}.* [1]

Point X_{486} is the Inner Vecten point.

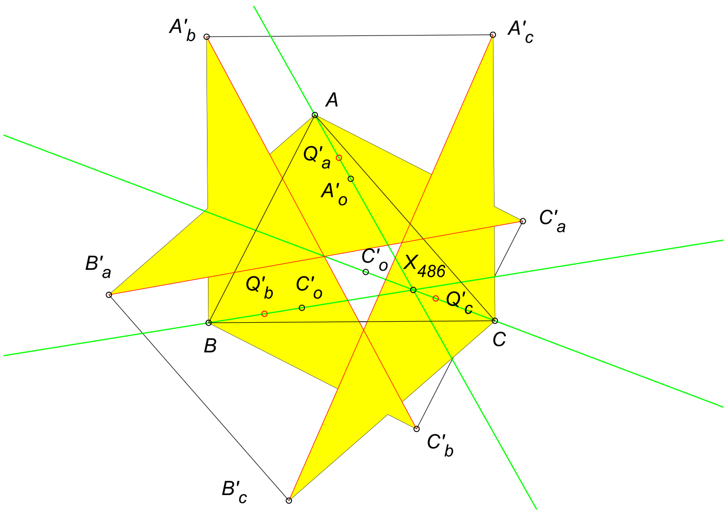

Figure 2. The Inner Vecten point

Proof. (See Figure 2) Let the squares are: $AB'_aB'_cC$, $CA'_cA'_bB$, and $BC'_bC'_aA$ with centers B'_o, A'_o, C'_o respectively.

Then $a'_b = b-(b-c)i$, $b'_c = c-(c-a)i$, $c'_a = a-(a-b)i$, $\bar{a}'_b = \frac{c-(b-c)i}{bc}$, $\bar{b}'_c = \frac{a-(c-a)i}{ca}$, $\bar{c}'_a = \frac{b-(a-b)i}{ab}$.

The coordinates of A'_o, B'_o, C'_o and its complex conjugates are

$$\begin{aligned} a'_o &= \frac{1}{2}(a'_b + c) = \frac{1}{2}((b+c)-(b-c)i) \qquad (6)\\ \bar{a}'_o &= \frac{(b+c)-(b-c)i}{2bc} \\ b'_o &= \frac{1}{2}(b'_c + a) = \frac{1}{2}((c+a)-(c-a)i) \\ \bar{b}'_o &= \frac{(c+a)-(c-a)i}{2ca} \end{aligned}$$

$$c'_o = \frac{1}{2}(c'_a + b) = \frac{1}{2}((a+b) - (a-b)i)$$
$$\bar{c}'_o = \frac{(a+b) - (a-b)i}{2ab}.$$

The line AA'_o, joining a' and a'_o has equation

$$0 = \begin{vmatrix} z & a & a'_o \\ \bar{z} & \bar{a} & \bar{a}'_o \\ 1 & 1 & 1 \end{vmatrix} = (\bar{a}'_o - \bar{a})z - (a'_o - a)\bar{z} + (a'_o\bar{a} - \bar{a}'_o a) \tag{7}$$

Lines BB'_o and CC'_o have equations

$$\begin{aligned} 0 &= (\bar{b}'_o - \bar{b})z - (b'_o - b)\bar{z} + (b'_o\bar{b} - \bar{b}'_o b) \\ &= \frac{\bar{b}'_o - \bar{b}}{b'_o - b}z - \bar{z} + \frac{b'_o\bar{b} - \bar{b}'_o b}{b'_o - b} \end{aligned} \tag{8}$$

$$\begin{aligned} 0 &= (\bar{c}'_o - \bar{c})z - (c'_o - c)\bar{z} + (c'_o\bar{c} - \bar{c}'_o c) \\ &= \frac{\bar{c}'_o - \bar{c}}{c'_o - c}z - \bar{z} + \frac{c'_o\bar{c} - \bar{c}'_o c}{c'_o - c} \end{aligned} \tag{9}$$

The intersection of the last two lines is given by

$$\begin{aligned} 0 &= \left(\frac{\bar{b}'_o - \bar{b}}{b'_o - b} - \frac{\bar{c}'_o - \bar{c}}{c'_o - c}\right)z + \left(\frac{b'_o\bar{b} - \bar{b}'_o b}{b'_o - b} - \frac{c'_o\bar{c} - \bar{c}'_o c}{c'_o - c}\right) \\ &= \frac{-2(a-b)(b-c)(c-a) + i(a(b-c)^2 + b(c-a)^2c(a-b)^2)}{2abc} \cdot z \\ &+ \frac{(a-b)(b-c)(c-a)(a+b+c) - i(a^2(b-c)^2 + b^2(c-a)^2 + c^2(a-b)^2)}{2abc} \end{aligned}$$

$$z = \frac{(a-b)(b-c)(c-a)(a+b+c) - i(a^2(b-c)^2 + b^2(c-a)^2 + c^2(a-b)^2)}{2(a-b)(b-c)(c-a) - i(a(b-c)^2 + b(c-a)^2 + c(a-b)^2)}$$

This is symmetric in a, b, c and means that the three lines AA'_o, BB'_o, CC'_o are concurrent. The point of concurrency is the Inner Vecten point X_{486}. □

Corollary 4. *Let the circumcircle of triangle ABC be the unit circle. The Inner Vecten point X_{486} has coordinates*

$$x_{486} = \frac{(a-b)(b-c)(c-a)(a+b+c) - i(a^2(b-c)^2 + b^2(c-a)^2 + c^2(a-b)^2)}{2(a-b)(b-c)(c-a) - i(a(b-c)^2 + b(c-a)^2 + c(a-b)^2)} \tag{10}$$

1.3. *The Orthopole.*

Lemma 5. *Let V and W be points of the plane. The orthogonal projection P_1 of a point P onto the line VW is given by*

$$p_1 = \frac{(\bar{w} - \bar{v})p + (w - v)\bar{p} - (w\bar{v} - \bar{w}v)}{2(\bar{w} - \bar{v})} \tag{11}$$

Proof. The line WV contains a point with coordinates p_1 if and only if

$$0=\begin{vmatrix} p_1 & \bar{p}_1 & 1\\ w & \bar{w} & 1\\ v & \bar{v} & 1\end{vmatrix}=\frac{\bar{w}-\bar{v}}{w-v}p_1-\bar{p}_1+\frac{w\bar{v}-\bar{w}v}{w-v}$$

$P_1P\perp WV$:

$$0=(p_1-p)(\bar{w}-\bar{v})+(\bar{p}_1-\bar{p})(w-v)=\frac{\bar{w}-\bar{v}}{w-v}p_1+\bar{p}_1-\frac{\bar{w}-\bar{v}}{w-v}p-\bar{p}$$

Adding the last two expressions, receive

$$2\frac{\bar{w}-\bar{v}}{w-v}p_1-\frac{\bar{w}-\bar{v}}{w-v}p-\bar{p}+\frac{w\bar{v}-\bar{w}v}{w-v}=0$$

Dividing by $2\frac{\bar{w}-\bar{v}}{w-v}$, we obtain p_1, as given in (11) above. □

Remark. [7, Lemma 3] In case V and W be points on the unit circle,

$$p_1=\frac{1}{2}(v+w+p-vw\bar{p})\,;\quad \bar{p}_1=\frac{1}{2vw}(v+w-p+vw\bar{p}) \tag{12}$$

Lemma 6. *Let the circumcircle of triangle ABC be the unit circle. Let EF be a arbitrary line in the plane. Let A_1 be the orthogonal projection of A onto line EF. The perpendicular from A_1 to BC has equation*

$$\frac{z}{bc}-\bar{z}+\frac{a(bc(\bar{e}-\bar{f})-(e-f))}{2bc(e-f)}+\frac{bc(\bar{e}-\bar{f})-(e-f)}{2abc(\bar{e}-\bar{f})}+\frac{(bc(\bar{e}-\bar{f})+(e-f))(e\bar{f}-\bar{e}f)}{2bc(e-f)(\bar{e}-\bar{f})}=0 \tag{13}$$

Proof. By Lemma 5, the coordinates a_1 of A_1 and its complex conjugate are

$$a_1=\frac{1}{2(\bar{e}-\bar{f})}\big((\bar{e}-\bar{f})a+(e-f)\frac{1}{a}-(e\bar{f}-\bar{e}f)\big)$$

$$\bar{a}_1=\frac{1}{2(e-f)}\big((\bar{e}-\bar{f})a+(e-f)\frac{1}{a}+(e\bar{f}-\bar{e}f)\big)$$

By (12), (Remark to Lemma 5), the coordinate a_2 of the orthogonal projection A_2 of A_1 onto BC, together with its conjugate, are

$$a_2=\frac{1}{2}(b+c+a_1-bc\bar{a}_1)$$

$$\bar{a}_2=\frac{1}{2bc}(b+c-a_1+bc\bar{a}_1)$$

The line A_1A_2 contains a point with coordinates z if and only if

$$0=\begin{vmatrix} z & a_1 & a_2\\ \bar{z} & \bar{a}_1 & \bar{a}_2\\ 1 & 1 & 1\end{vmatrix}=\frac{\bar{a}_1-\bar{a}_2}{a_1-a_2}z-\bar{z}+\frac{a_1\bar{a}_2-\bar{a}_1a_2}{a_1-a_2}=\frac{1}{bc}z-\bar{z}$$

$$+\frac{(a^2(\bar{e}-\bar{f})+(e-f))(bc(\bar{e}-\bar{f})-(e-f))+a(e\bar{f}-\bar{e}f)(bc(\bar{e}-\bar{f})+(e-f))}{2abc(e-f)(\bar{e}-\bar{f})} \tag{14}$$

This is equal to (13). □

Theorem 7 (*Orthopole*, [2, p 247, Theorem 406]). *If perpendiculars are dropped on any line from the vertices of a triangle, the perpendiculars to the opposite sides from their feet are concurrent at a point called the orthopole of the point.*

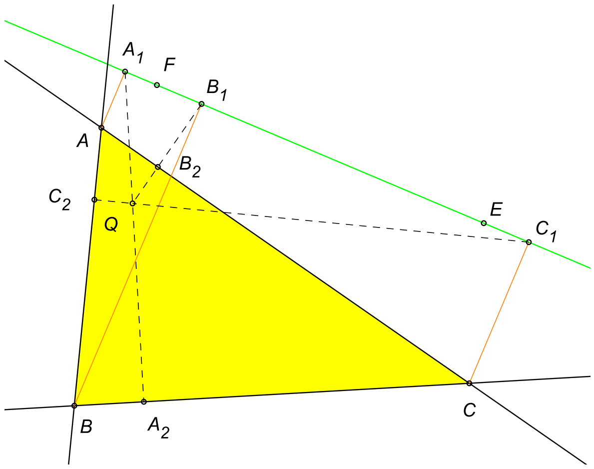

Figure 3. The Orthopole

Proof. Let EF be a arbitrary line of the plane. Let A_1, B_1, C_1 be the orthogonal projections from vertexes A, B, C respectively to the line EF (Figure 3).

We consider the construction in Lemma 6 beginning with all three vertices of triangle ABC. This results in the three lines

$$\frac{z}{bc}-\bar{z}+\frac{a(bc(\bar{e}-\bar{f})-(e-f))}{2bc(e-f)}+\frac{bc(\bar{e}-\bar{f})-(e-f)}{2abc(\bar{e}-\bar{f})}+\frac{(bc(\bar{e}-\bar{f})+(e-f))(e\bar{f}-\bar{e}f)}{2bc(e-f)(\bar{e}-\bar{f})}=0$$

$$\frac{z}{ca}-\bar{z}+\frac{b(ca(\bar{e}-\bar{f})-(e-f))}{2ca(e-f)}+\frac{ca(\bar{e}-\bar{f})-(e-f)}{2abc(\bar{e}-\bar{f})}+\frac{(ca(\bar{e}-\bar{f})+(e-f))(e\bar{f}-\bar{e}f)}{2ca(e-f)(\bar{e}-\bar{f})}=0$$

$$\frac{z}{ab}-\bar{z}+\frac{c(ab(\bar{e}-\bar{f})-(e-f))}{2ab(e-f)}+\frac{ab(\bar{e}-\bar{f})-(e-f)}{2abc(\bar{e}-\bar{f})}+\frac{(ab(\bar{e}-\bar{f})+(e-f))(e\bar{f}-\bar{e}f)}{2ab(e-f)(\bar{e}-\bar{f})}=0$$

The intersection of the last two lines is

$$\frac{z(b-c)}{abc}+\frac{abc(b-c)(\bar{e}-\bar{f})}{2abc(e-f)}-\frac{(b^2-c^2)(e-f)}{2abc(e-f)}-\frac{a(b-c)(\bar{e}-\bar{f})}{2abc(\bar{e}-\bar{f})}+\frac{(b-c)(e-f)(e\bar{f}-\bar{e}f)}{2abc(e-f)(\bar{e}-\bar{f})}=0$$

Multiplying by $\frac{abc}{b-c}$, we obtain

$$z=\frac{1}{2}(a+b+c)-\frac{abc(\bar{e}-\bar{f})}{2(e-f)}-\frac{e\bar{f}-\bar{e}f}{2(\bar{e}-\bar{f})}$$

Note that this is symmetric in a, b, c. This means that the three perpendiculars from A_1 to BC, B_1 to CA, and C_1 to AB are concurrent. The point of concurrency is the orthopole Q of the line EF with respect to triangle ABC. $\square$

Corollary 8. *Let the circumcircle of triangle ABC be the unit circle. The orthopole Q of the line EF with respect to triangle ABC has complex coordinate and conjugate*

$$q = \frac{1}{2}(a+b+c) - \frac{abc(\bar{e}-\bar{f})}{2(e-f)} - \frac{e\bar{f}-\bar{e}f}{2(\bar{e}-\bar{f})} \tag{15}$$

$$\bar{q} = \frac{ab+bc+ca}{2abc} - \frac{e-f}{2abc(\bar{e}-\bar{f})} + \frac{e\bar{f}-\bar{e}f}{2(e-f)}$$

2. Orthopoles and flank lines

Theorem 9. *Erect a square outwardly from each side of triangle ABC: AC_aC_bB, BA_bA_cC and CB_cB_aA with centers C_o, A_o, B_o respectively. For flank lines B_aC_a, C_bA_b, A_cB_c construct the orthopoles Q_a, Q_b, Q_c respectively. The lines AQ_a, BQ_b, CQ_c concur in Vecten point X_{485}.*

The proof of the theorem relies on the following lemma.

Lemma 10. *The orthopole Q_a of the flank line B_aC_a with respect to triangle ABC and points A, A_o are collinear.*

Proof. (See Figure 1) With no loss of generality we shall say that the circumcircle of triangle ABC is the union circle. $b_a = a + (c-a)i$, $c_a = a + (a-b)i$, $\bar{b}_a = \frac{c+(c-a)i}{ac}$, $\bar{c}_a = \frac{b+(a-b)i}{ab}$

By Corollary 8 find the orthopole Q_a as substitute b_a, c_a for e, f in (15).

$$q_a = \frac{1}{2}(a+b+c) - \frac{abc(\bar{b}_a-\bar{c}_a)}{2(b_a-c_a)} - \frac{b_a\bar{c}_a-\bar{b}_ac_a}{2(\bar{b}_a-\bar{c}_a)}$$

$$q_a = \frac{1}{2}(a+b+c) - \frac{a(b+c)-2bc}{2(2a-b-c)} + \frac{b(c-a)^2+c(a-b)^2-i(a-b)(b-c)(c-a)}{2(a(b+c)-2bc)} \tag{16}$$

$$\bar{q}_a = \frac{ab+bc+ca}{2abc} + \frac{2a-b-c}{2(a(b+c)-2bc)} - \frac{b(c-a)^2+c(a-b)^2-i(a-b)(b-c)(c-a)}{2abc(2a-b-c)} \tag{17}$$

The points A, A_o, Q_a are collinear if and only if ([5, p. 65], [6, p. 37])

$$\frac{q_a-a}{a_o-a} = \eta \in \mathbb{R}^* \text{ i.e. } \frac{q_a-a}{a_o-a} = \frac{\bar{q}_a-\bar{a}}{\bar{a}_o-\bar{a}}$$

Use (16), (17), (1) and $\bar{a} = 1/a$.

$$\begin{aligned}&(q_a-a)(\bar{a}_o-\bar{a}) - (\bar{q}_a-\bar{a})(a_o-a)\\ =&\frac{(a-b)(c-a)(bc-a^2)(b-c)^2(i^2+1)}{2abc(b+c-2a)(2bc-ab-ca)} = 0\end{aligned}$$

□

Proof of Theorem 9. (See Figure 1) By Lemma 10 applied to all three vertices of triangle ABC: $Q_a \in AA_o$, $Q_b \in BB_o$ and $Q_c \in CC_o$. But from Theorem 1, the lines AA_o, BB_o, CC_o concur in the Vecten point X_{485}. $\square$

Theorem 11. *Erect a square inwardly from each side of triangle ABC : $BC'_bC'_aA$, $CA'_cA'_bB$ and $AB'_aB'_cC$ with centers C'_o, A'_o, B'_o respectively. For flank lines $B'_aC'_a$, $C'_bA'_b$, $A'_cB'_c$ construct the orthopoles Q'_a, Q'_b, Q'_c respectively. The lines AQ'_a, BQ'_b, CQ'_c concur in Inner Vecten point X_{486}.*

Lemma 12. *The orthopole Q'_a of the flank line $B'_aC'_a$ with respect to triangle ABC and points A, A'_o are collinear. (See Figure 2)*

Proof. The proof is the same as the proof of Lemma 10.

With no loss of generality we shall say that the circumcircle of triangle ABC is the union circle. $b'_a = a-(c-a)i$, $c'_a = a-(a-b)i$, $\bar{b}'_a = \frac{c-(c-a)i}{ca}$, $\bar{c}'_a = \frac{b-(a-b)i}{ab}$

$$q'_a = \frac{1}{2}(a+b+c) - \frac{a(b+c)-2bc}{2(2a-b-c)} + \frac{b(c-a)^2 + c(a-b)^2 + i(a-b)(b-c)(c-a)}{2(a(b+c)-2bc)} \tag{18}$$

$\square$

Proof of Theorem 11. (See Figure 2) By Lemma 12 applied to all three vertices of triangle ABC: $Q'_a \in AA'_o$, $Q'_b \in BB'_o$ and $Q'_c \in CC'_o$. But from Theorem 3, the lines AA'_o, BB'_o, CC'_o concur in the Inner Vecten point X_{486}. $\square$

Lemma 13. [4, p. 105] *Erect squares outwardly from sides of triangle ABC: AC_aC_bB, BA_bA_cC and CB_cB_aA with centers C_o, A_o, B_o.*

Erect a square $B_aB_{a1}C_{a1}C_a$ with center A_{o1} outwardly from B_aC_a. The points A_{o1}, A_o and A are collinear.

Proof. With no loss of generality, we can say that the circumcircle $\mathcal{C}$ of triangle ABC is the union circle. $b_a = a + (c-a)i$, $c_a = a + (a-b)i$, $\bar{b}_a = \frac{c+(c-a)i}{ac}$, $\bar{c}_a = \frac{b+(a-b)i}{ab}$

$$\begin{aligned}
b_{a1} &= |b_a + (b_a - c_a)i = 3a - b - c + (c-a)i \\
\bar{b}_{a1} &= \frac{3bc - ca - ab + (c-a)i}{abc} \\
a_{o1} &= \frac{1}{2}(c_a + b_{a1}) = \frac{1}{2}(4a - b - c - (b-c)i) \\
\bar{a}_{o1} &= \frac{4bc - ca - ab - a(b-c)i}{2abc}.
\end{aligned}$$

The points A, A_o, A_{o1} are collinear if and only if ([5, p. 65], [6, p. 37])

$$\frac{a_{o1}-a}{a_o-a} = \eta \in \mathbb{R}^* \text{ i.e. } \frac{a_{o1}-a}{a_o-a} = \frac{\bar{a}_{o1}-\bar{a}}{\bar{a}_o-\bar{a}}$$

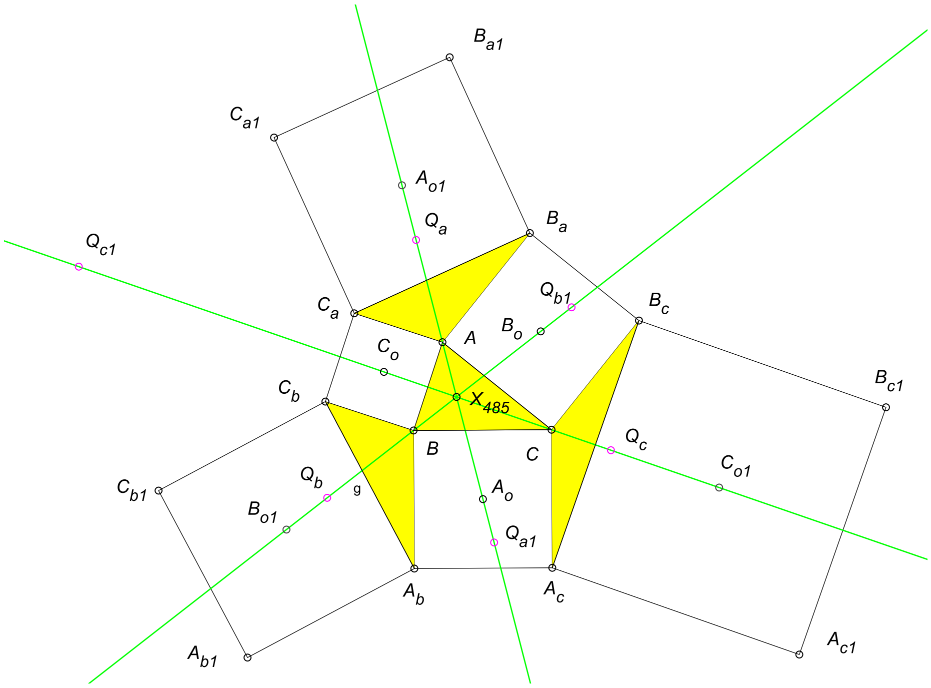

Figure 4.

$$
\begin{aligned}
&(a_{o1}-a)(\bar{a}_o-\bar{a})-(a_o-a)(\bar{a}_{o1}-\bar{a})\\
=&\frac{2a-b-c-(b-c)i}{2}\cdot\frac{a(b+c)+a(b-c)i-2bc}{2abc}\\
-&\frac{2bc-ca-ab-a(b-c)i}{2abc}\cdot\frac{b+c-2a+(b-c)i}{2}=0
\end{aligned}
$$

□

Theorem 14. *Erect a square outwardly from each side of triangle ABC: AC_aC_bB, BA_bA_cC and CB_cB_aA. Construct the orthopoles Q_{a1} for BC with respect to flank triangle AB_aC_a; Q_{b1} for CA with respect to flank triangle BC_bA_b; Q_{c1} for AB with respect to flank triangle CA_cB_c. The lines AQ_{a1}, BQ_{b1}, CQ_{c1} concur in Vecten point X_{485}.*

Proof. (See Figure 4) Let C_o, A_o, B_o be centers of the squares AC_aC_bB, BA_bA_cC, and CB_cB_aA respectively. Erect squares $B_aB_{a1}C_{a1}C_a$, $C_bC_{b1}A_{b1}A_b$, $A_cA_{c1}B_{c1}B_c$ outwardly from B_aC_a, C_bA_b, A_cB_c with centers A_{o1}, B_{o1}, C_{o1} respectively.

Triangle ABC is a flank triangle for triangle AB_aC_a. From Lemma 10 the orthopole Q_{a1} of the flank line BC with respect to triangle AB_aC_a and points A, A_{o1} are collinear. From Lemma 14 points A_{o1}, A_o and A are collinear. Follow Q_{a1}, A_{o1}, A, A_o are collinear.

By analogy Q_{b1}, B_{o1}, B, B_o and Q_{c1}, C_{o1}, C, C_o are collinear. From Theorem 1 the last three lines concur in the Vecten point X_{485}. □

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