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Geometric Inequalities in Pedal Quadrilaterals

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Abstract. The aim of this paper is to investigate the general properties of the pedal quadrilateral of a point P with respect to a convex and closed quadrilateral $ABCD$. In particular, we present an analogue of Erdős-Mordell Inequality, stating that for any triangle ABC and a point P inside ABC, the sum of the distances from P to the sides is less than or equal to half of the sum of the distances from P to the vertices of ABC, for an inscribed quadrilateral.

1. Introduction

Let ABC be a triangle and P be a point in plane. If the feet of the perpendiculars drawn from P to sides of the triangle $[BC]$, $[CA]$, $[AB]$ are respectively X, Y and Z, then triangle XYZ is called the pedal triangle of P with respect to ABC and P is called the pedal point ([3, 7]). If the point P lies on the circumcircle of the triangle ABC, then the points X, Y and Z are collinear. The line passing through the points X, Y and Z is known as the *Simson line* of P ([3]).

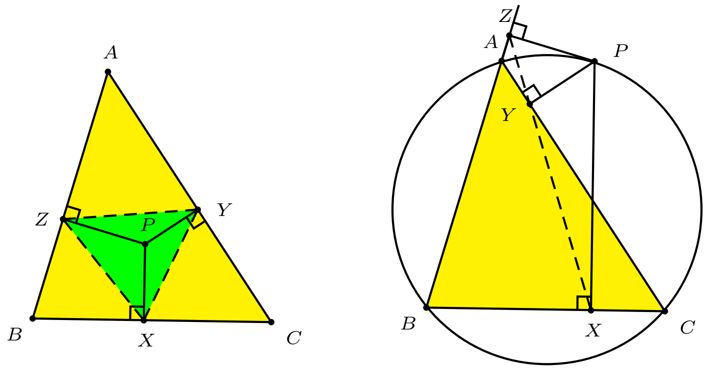

Figure 1

A similar notion has been generalized for polygons with n sides ([3]). Let $A_1A_2A_3 \cdots A_n$ be a polygon and P be a point inside $A_1A_2A_3 \cdots A_n$. If the feet of the perpendiculars from P to the line segments $[A_1A_2]$, $[A_2A_3]$, ..., $[A_nA_1]$ are respectively H_1, H_2, ..., H_n, then the polygon $H_1H_2 \cdots H_n$ is called the *pedal polygon* of P with respect to $A_1A_2A_3 \cdots A_n$ and P is called the pedal point ([3]). Although there are lots of investigations on pedal triangles, so little investigations have been studied on pedal quadrilaterals in the literature ([3, 5, 6, 7, 9, 11]).

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In this paper, we study on the general properties of the pedal quadrilateral of a point P with respect to a convex and closed quadrilateral $ABCD$. We investigate some interesting geometric inequalities on pedal quadrilaterals. In particular, we state an analogue of Erdős-Mordell Inequality ([1, 2, 4, 8, 10]) for quadrilaterals.

2. Geometric inequalities in pedal quadrilaterals

Proposition 1. *Let $ABCD$ be an inscribed quadrilateral and P be a point inside $ABCD$. If the feet of perpendiculars from P to the line segments $[AB]$, $[BC]$, $[CD]$ and $[AD]$ are respectively K, M, N, L and the radius of the circumcircle of $ABCD$ is R then,*

$$|KM| = \tfrac{|AC|\cdot|PB|}{2R}, \qquad |KL| = \tfrac{|BD|\cdot|PA|}{2R},$$

$$|MN| = \tfrac{|BD|\cdot|PC|}{2R}, \qquad |LN| = \tfrac{|AC|\cdot|PD|}{2R}.$$

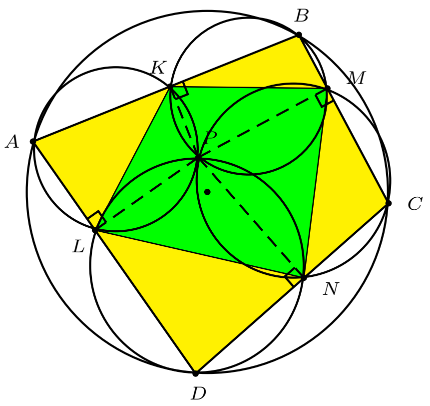

Figure 2

Proof. As seen in Figure 2 and since $\angle BKP = \angle PLD = \angle PNC = \angle PMB = 90°$, the quadrilaterals $BKPM$, $PLDN$, $PNCM$ and $PLAK$ are inscribed quadrilaterals. By the law of sines, we have

$$|PB| = \frac{KM}{\sin KPM} \quad \text{and} \quad 2R = \frac{|AC|}{\sin(\pi - \angle KPM)} = \frac{|AC|}{\sin KPM}.$$

Hence, we get

$$|KM| = \frac{|AC|\cdot|PB|}{2R}.$$

It can be easily seen that

$$|KL| = \frac{|BD|\cdot|PA|}{2R}, \qquad |MN| = \frac{|BD|\cdot|PC|}{2R}, \qquad |LN| = \frac{|AC|\cdot|PD|}{2R}.$$

□

Proposition 2. *Let $ABCD$ be a quadrilateral and P be a point inside $ABCD$.If the feet of perpendiculars from P to the line segments $[AB]$, $[BC]$, $[CD]$ and $[AD]$ are respectively A_1, B_1, C_1, D_1, then*

$$r_1 + r_2 + r_3 + r_4 \leq \frac{M \cdot C}{2\sqrt{2}},$$

where r_1, r_2, r_3 and r_4 are respectively the radii of the incircles of the triangles AA_1D_1, A_1BB_1, CB_1C_1, DC_1D_1, and

$$M = \max\left\{\frac{1}{\sqrt{1-\cos A}+\sqrt{2}}, \frac{1}{\sqrt{1-\cos B}+\sqrt{2}}, \frac{1}{\sqrt{1-\cos C}+\sqrt{2}}, \frac{1}{\sqrt{1-\cos D}+\sqrt{2}}\right\},$$

and C is the perimeter of the quadrilateral $ABCD$. The equality holds if $ABCD$ is chosen as a square and P is chosen at the center of the square.

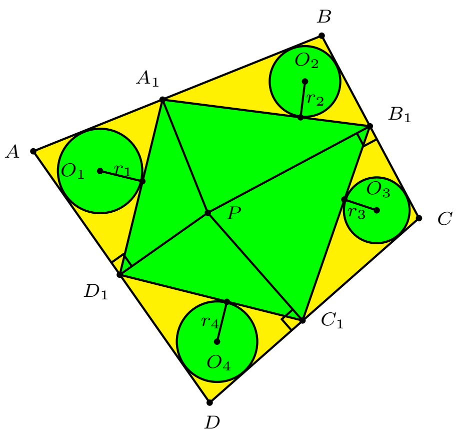

Figure 3

Proof. By the law of cosines, we have

$$|A_1B_1| = \sqrt{|A_1B|^2 + |BB_1|^2 - 2|A_1B||BB_1|\cos B}. \tag{1}$$

If $S_{A_1BB_1}$ is the area of the triangle A_1BB_1, then

$$S_{A_1BB_1} = \frac{|A_1B| \cdot |BB_1| \sin B}{2}. \tag{2}$$

Since $2S_{A_1BB_1} = (|A_1B| + |BB_1| + |A_1B_1|)r_2$, we get

$$r_2 = \frac{2S_{A_1BB_1}}{|A_1B| + |BB_1| + |A_1B_1|}. \tag{3}$$

By the equalities (2) and (3),

$$r_2 = \frac{|A_1B| \cdot |BB_1| \sin B}{|A_1B| + |BB_1| + |A_1B_1|}. \tag{4}$$

It follows from Arithmetic-Geometric Mean Inequality that

$$|A_1B|^2 + |BB_1|^2 \geq 2 \cdot |A_1B| \cdot |BB_1|. \tag{5}$$

If the inequalities (1) and (5) are considered together, then

$$\begin{aligned}|A_1B_1| =&\ \sqrt{|A_1B|^2+|BB_1|^2-2|A_1B|\cdot|BB_1|\cos B}\\ \geq&\ \sqrt{2|A_1B|\cdot|BB_1|-2|A_1B|\cdot|BB_1|\cos B}\\ \geq&\ \sqrt{2}\,\sqrt{|A_1B|\cdot|BB_1|}\sqrt{1-\cos B} \qquad (6)\end{aligned}$$

It is also obvious from Arithmetic-Geometric Mean Inequality that

$$|A_1B|+|BB_1| \geq 2\sqrt{|A_1B|\cdot|BB_1|}. \qquad (7)$$

Therefore, by the inequalities (6) and (7), we observe that

$$|A_1B|+|BB_1|+|A_1B_1| \geq 2\sqrt{|A_1B|\cdot|BB_1|}+\sqrt{2}\,\sqrt{|A_1B|\cdot|BB_1|}\sqrt{1-\cos B}. \qquad (8)$$

Hence, it follows from the inequalities (4) and (8) that

$$\begin{aligned}r_2 \leq&\ |A_1B|\cdot|BB_1|\frac{\sin B}{\sqrt{2}\cdot\sqrt{|A_1B|\cdot|BB_1|}\sqrt{1-\cos B}+2\sqrt{|A_1B|\cdot|BB_1|}}\\ \leq&\ |A_1B|\cdot|BB_1|\cdot\frac{\sin B}{\sqrt{2}\sqrt{|A_1B|\cdot|BB_1|}\sqrt{1-\cos B}+\sqrt{2})}\\ \leq&\ \sqrt{|A_1B|\cdot|BB_1|}\cdot\frac{\sin B}{\sqrt{2}(\sqrt{1-\cos B}+\sqrt{2})}\\ \leq&\ \frac{|A_1B|+|BB_1|}{2\sqrt{2}(\sqrt{1-\cos B}+\sqrt{2})} \qquad (9)\end{aligned}$$

Let $M = \max\left\{\frac{1}{\sqrt{1-\cos A}+\sqrt{2}}, \frac{1}{\sqrt{1-\cos B}+\sqrt{2}}, \frac{1}{\sqrt{1-\cos C}+\sqrt{2}}, \frac{1}{\sqrt{1-\cos D}+\sqrt{2}}\right\}$. By the inequality (9),

$$r_2 \leq \frac{M\cdot(|A_1B|+|BB_1|)}{2\sqrt{2}}. \qquad (10)$$

Similarly, we get

$$\left.\begin{aligned} r_1 &\leq \tfrac{M\cdot(|A_1A|+|AD_1|)}{2\sqrt{2}},\\ r_3 &\leq \tfrac{M\cdot(|C_1C|+|CB_1|)}{2\sqrt{2}},\\ r_4 &\leq \tfrac{M\cdot(|C_1D|+|DD_1|)}{2\sqrt{2}}.\end{aligned}\right\} \qquad (11)$$

If the inequalities (10) and (11) are added side by side, it is easy to see that

$$r_1+r_2+r_3+r_4 \leq \frac{M\cdot C}{2\sqrt{2}}$$

where C is the perimeter of the quadrilateral $ABCD$.

Now suppose that $ABCD$ is a square with side length $2a$ and P is at the center of the $ABCD$ (Figure 4). If the area of the triangle D_1AA_1 is $S_{D_1AA_1}$ then,

$$S_{D_1AA_1} = \frac{2a+a\sqrt{2}}{2}r_1 = \frac{a^2}{2}.$$

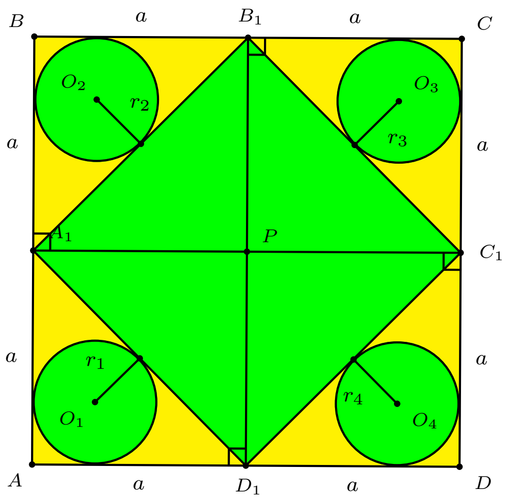

Figure 4

Hence, we get $r_1 = \frac{a}{2+\sqrt{2}}$. Since $r_1 = r_2 = r_3 = r_4$, $C = 8a$ and $M = \frac{1}{\sqrt{2}+1}$, then the following equality holds:

$$r_1 + r_2 + r_3 + r_4 = 4 \cdot \frac{a}{2+\sqrt{2}} = \frac{8a \cdot \left(\frac{1}{\sqrt{2}+1}\right)}{2\sqrt{2}} = \frac{C \cdot M}{2\sqrt{2}}.$$

□

Proposition 3. *Let $A_1A_2A_3 \cdot A_n$ be a polygon and P be a point inside $A_1A_2A_3 \cdots A_n$. If the feet of perpendiculars from P to the line segments $[A_1A_2]$, $[A_2A_3]$, $\ldots$, $[A_nA_1]$ are respectively H_1, H_2, $\ldots$, H_n, then*

$$r_1 + r_2 + \cdots + r_n \leq \frac{M \cdot C}{2\sqrt{2}}$$

where r_1, r_2, $\ldots$, r_n are respectively the radii of the incircles of the triangles $A_1H_1H_n$, $H_1A_2H_2, \ldots$, $H_{n-1}A_nH_n$, and

$$M = \max\left\{\frac{1}{\sqrt{1-\cos A_1}+\sqrt{2}}, \frac{1}{\sqrt{1-\cos A_2}+\sqrt{2}}, \ldots, \frac{1}{\sqrt{1-\cos A_n}+\sqrt{2}}\right\}$$

and C is the perimeter of the polygon $A_1A_2 \cdots A_n$. The equality holds if $A_1A_2 \cdots A_n$ is chosen as a square and P is chosen as the centroid of the square.

This can be easily shown as in Proposition 2.

Proposition 4. *Let $ABCD$ be a circumscribed quadrilateral with area S. If the center and the radius of the incircle of $ABCD$ are respectively the point P and r, and the feet of perpendiculars from P to the line segments $[AB]$, $[BC]$, $[CD]$ and*

$[AD]$ are respectively K, L, M and N, then

$$\frac{S_{\text{pedal}}}{S} \leq 1 - \frac{1}{4r}\sqrt{a^2+b^2+c^2+d^2},$$

where $a = |BL|$, $b = |LC|$, $c = |MD|$, $d = |AN|$, and S_{pedal} is the area of the pedal quadrilateral $KLMN$. The equality holds if $ABCD$ is a square.

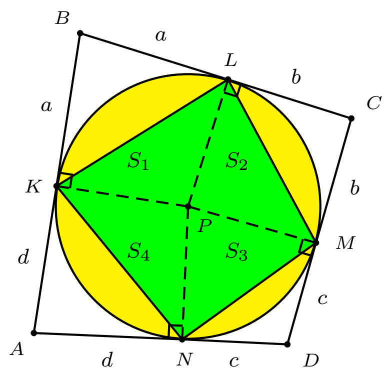

Figure 5

Proof. As seen in Figure 5, if S_1, S_2, S_3 and S_4 are respectively the areas of triangles KPL, LPM, MPN and NPK, then we get

$$\begin{aligned} S_1 &= a \cdot r - \frac{a^2 \sin B}{2}, \\ S_2 &= b \cdot r - \frac{b^2 \sin C}{2}, \\ S_3 &= c \cdot r - \frac{c^2 \sin D}{2}, \\ S_4 &= d \cdot r - \frac{d^2 \sin A}{2}. \end{aligned}$$

Hence, it is obvious that

$$S_1+S_2+S_3+S_4 = r(a+b+c+d) - \frac{1}{2}(a^2 \sin B + b^2 \sin C + c^2 \sin D + d^2 \sin A).$$

Since we also have $S = (a+b+c+d)r$, we get

$$\begin{aligned} \frac{S_{\text{pedal}}}{S} &= \frac{S_1+S_2+S_3+S_4}{S} \\ &= 1 - \frac{a^2 \sin B + b^2 \sin C + c^2 \sin D + d^2 \sin A}{2r(a+b+c+d)}. \end{aligned} \tag{12}$$

By the Cauchy-Schwarz Inequality, we get

$$\begin{aligned} a+b+c+d &= a\cdot 1+b\cdot 1+c\cdot 1+d\cdot 1 \\ &\le \sqrt{a^2+b^2+c^2+d^2}\cdot\sqrt{1^2+1^2+1^2+1^2} \\ &= 2\sqrt{a^2+b^2+c^2+d^2}. \end{aligned}$$

Thus,

$$-\frac{1}{a+b+c+d} \le -\frac{1}{2\sqrt{a^2+b^2+c^2+d^2}}.$$

By the equality (12),

$$\begin{aligned} \frac{S_{\text{pedal}}}{S} &= 1-\frac{a^2\sin B+b^2\sin C+c^2\sin D+d^2\sin A}{2r(a+b+c+d)} \\ &\le 1-\frac{a^2+b^2+c^2+d^2}{4r\sqrt{a^2+b^2+c^2+d^2}} \\ &= 1-\frac{1}{4r}\sqrt{a^2+b^2+c^2+d^2}. \end{aligned}$$

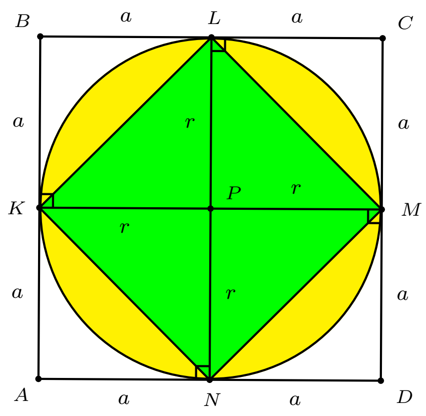

Figure 6

Now suppose that $ABCD$ is a square and $|AB|=|BC|=|CD|=|AD|=2a$. As seen in Figure 6, since $r=a$, $S=4a^2$, $S_{\text{pedal}}=2a^2$, we have

$$\frac{S_{\text{pedal}}}{S}=\frac{2a^2}{4a^2}=\frac{1}{2}=1-\frac{1}{4a}\sqrt{a^2+a^2+a^2+a^2}=1-\frac{1}{4r}\sqrt{a^2+b^2+c^2+d^2}.$$

□

Proposition 5. *Let P be a point inside a polygon $A_1A_2A_3\cdots A_n$ where $|A_1A_2|=a_1$, $|A_2A_3|=a_2$, ..., $|A_nA_1|=a_n$. If the lengths of the perpendiculars from P to the sides of the polygon $A_1A_2A_3\cdots A_n$ are respectively h_1, h_2, ..., h_n, then*

$$\frac{S}{C^2}\ge\frac{1}{2}\cdot\frac{1}{\frac{a_1}{h_1}+\cdots+\frac{a_n}{h_n}}$$

where S and C are respectively the area and the perimeter of $A_1A_2A_3\cdots A_n$. The equality holds if $a_1 = a_2 = \cdots = a_n = a$ and $h_1 = h_2 = \cdots = h_n = h$.

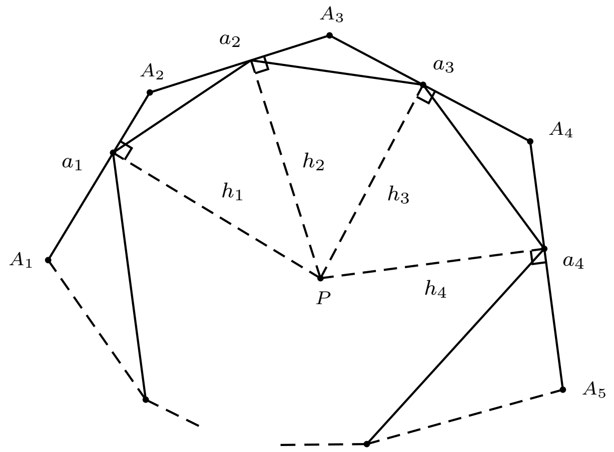

Figure 7

Proof. As seen in Figure 7, it is obvious that $2S = a_1h_1 + a_2h_2 + \cdots + a_nh_n$. By the Cauchy-Schwarz Inequality, we get

$$(a_1h_1+a_2h_2+\cdots+a_nh_n)\left(\frac{a_1}{h_1}+\frac{a_2}{h_2}+\cdots+\frac{a_n}{h_n}\right) \geq (\sqrt{a_1^2}+\sqrt{a_2^2}+\cdots+\sqrt{a_n^2})^2.$$

Since

$$2S\left(\frac{a_1}{h_1}+\frac{a_2}{h_2}+\cdots+\frac{a_n}{h_n}\right) \geq (\sqrt{a_1^2}+\sqrt{a_2^2}+\cdots+\sqrt{a_n^2})^2$$

we have

$$2S \geq \frac{(a_1+a_2+\cdots+a_n)^2}{\frac{a_1}{h_1}+\frac{a_2}{h_2}+\cdots+\frac{a_n}{h_n}} = \frac{C^2}{\frac{a_1}{h_1}+\frac{a_2}{h_2}+\cdots+\frac{a_n}{h_n}}.$$

Hence,

$$\frac{S}{C^2} \geq \frac{1}{2}\cdot\frac{1}{\frac{a_1}{h_1}+\frac{a_2}{h_2}+\cdots+\frac{a_n}{h_n}}.$$

Now suppose that $a_1 = a_2 = \cdots = a_n = a$, and $h_1 = h_2 = \cdots = h_n = h$. Since,

$$S = n\cdot\frac{ah}{2}, \qquad C = na,$$

we get

$$\frac{S}{C^2} = \frac{nah}{2n^2a^2} = \frac{h}{2na} = \frac{1}{2}\cdot\frac{1}{\frac{a}{h}+\frac{a}{h}+\cdots+\frac{a}{h}} = \frac{1}{2}\cdot\frac{1}{\frac{a_1}{h_1}+\frac{a_2}{h_2}+\cdots+\frac{a_n}{h_n}}.$$

□

Proposition 6. *Let P be a point inside a polygon $A_1A_2A_3\cdots A_n$ where $|A_1A_2| = a_1$, $|A_2A_3| = a_2, \ldots, |A_nA_1| = a_n$. If the lengths of the perpendiculars from P to the sides of the polygon $A_1A_2A_3\cdots A_n$ are respectively $h_1, h_2, \ldots, h_n$, then*

$$\frac{1}{a_1h_1} + \frac{1}{a_2h_2} + \cdots + \frac{1}{a_nh_n} \geq \frac{n^2}{2S}$$

where S is the area of $A_1A_2A_3\cdots A_n$. The equality holds if $a_1 = a_2 = \cdots = a_n = a$ and $h_1 = h_2 = \cdots = h_n = h$.

Proof. By the Cauchy-Schwarz Inequality, we have

$$(a_1h_1+a_2h_2+\cdots+a_nh_n)\left(\frac{1}{a_1h_1} + \frac{1}{a_2h_2} + \cdots + \frac{1}{a_nh_n}\right) \geq (\underbrace{1+1+\cdots+1}_{n \text{ times}})^2.$$

Since $2S = a_1h_1 + a_2h_2 + \cdots + a_nh_n$, it is easy to see that

$$\frac{1}{a_1h_1} + \frac{1}{a_2h_2} + \cdots + \frac{1}{a_nh_n} \geq \frac{n^2}{2S}.$$

Now suppose that $a_1 = a_2 = \cdots = a_n = a$ and $h_1 = h_2 = \cdots = h_n = h$. Since $S = n \cdot \frac{ah}{2}$, we have

$$\frac{1}{a_1h_1} + \frac{1}{a_2h_2} + \cdots + \frac{1}{a_nh_n} = n \cdot \frac{1}{ah} = \frac{n^2}{nah} = \frac{n^2}{2S}.$$

□

Theorem 7. *Let $ABCD$ be an inscribed quadrilateral and P be a point inside $ABCD$. If the feet of perpendiculars from P to the line segments $[AB]$, $[BC]$, $[CD]$ and $[AD]$ are A_1, B_1, C_1, D_1 respectively, then*

$$|PA| + |PB| + |PC| + |PD| > 4\sqrt[4]{|PA_1| \cdot |PA_2| \cdot |PA_3| \cdot |PA_4|}.$$

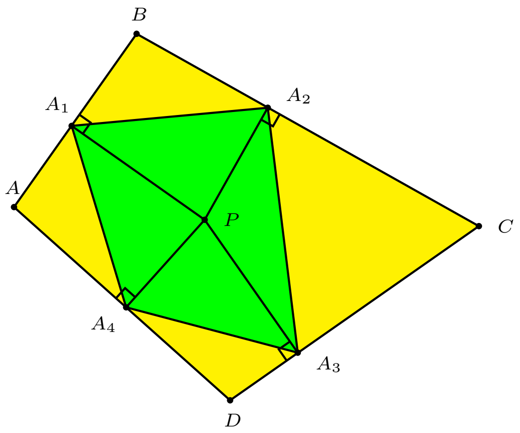

Figure 8

Proof. Let $|PA_1| = h_1$, $|PA_2| = h_2$, $|PA_3| = h_3$ and $|PA_4| = h_4$. Then by the law of cosines, we have

$$\begin{aligned}
|A_1A_2| &= \sqrt{h_1^2 + h_2^2 - 2h_1h_2\cos(\pi - B)},\\
|A_2A_3| &= \sqrt{h_2^2 + h_3^2 - 2h_2h_3\cos(\pi - C)},\\
|A_3A_4| &= \sqrt{h_3^2 + h_4^2 - 2h_3h_4\cos(\pi - D)},\\
|A_4A_1| &= \sqrt{h_4^2 + h_1^2 - 2h_1h_4\cos(\pi - A)}.
\end{aligned}$$

Hence,

$$\begin{aligned}
|A_1A_2|^2 &= h_1^2 + h_2^2 - 2h_1h_2\cos(\pi - B)\\
&= h_1^2 + h_2^2 - 2h_1h_2\cos(A + C + D - \pi)\\
&= h_1^2 + h_2^2 + 2h_1h_2\cos(A + C + D)\\
&= h_1^2 + h_2^2 + 2h_1h_2\cos(A + C)\cos D - 2h_1h_2\sin(A + C)\sin D\\
&= (h_2\sin D - h_1\sin(A + C))^2 + (h_2\cos D + h_1\cos(A + C))^2.
\end{aligned}$$

Since $A + C = \pi$, we have $|A_1A_2|^2 = (h_2\sin D)^2 + (h_2\cos D - h_1)^2$.

Now suppose that $(h_2\cos D - h_1)^2 = 0$. Then, $\cos D = \frac{h_1}{h_2}$. Since $\angle A_1PA_2 = \angle D$, by the law of cosines,

$$\begin{aligned}
|A_1A_2|^2 &= h_1^2 + h_2^2 - 2h_1h_2\cos D\\
&= h_1^2 + h_2^2 - 2h_1h_2 \cdot \frac{h_1}{h_2}\\
&= h_2^2 - h_1^2.
\end{aligned}$$

Hence, we get $h_2^2 = |A_1A_2|^2 + h_1^2$, implying that $\angle PA_1A_2 = 90°$, which is a contradiction. Therefore, $(h_2\cos D - h_1)^2 \neq 0$. Then, it is easy to verify that

$$|A_1A_2| > h_2\sin D \tag{13}$$

Similarly, we have

$$\begin{aligned}
|A_2A_3| &> h_3\sin A &\quad (14)\\
|A_3A_4| &> h_4\sin B &\quad (15)\\
|A_1A_4| &> h_1\sin C &\quad (16)
\end{aligned}$$

Since PA_1BA_2, PA_2CA_3, PA_3DA_4 and PA_4AA_1 are inscribed quadrilaterals, if we apply the law of sines to the triangles AA_1A_4, BA_1A_2, CA_2A_3, DA_3A_4, then we respectively get,

$$|PA| = \frac{|A_1A_4|}{\sin A} \tag{17}$$

$$|PB| = \frac{|A_1A_2|}{\sin B} \tag{18}$$

$$|PC| = \frac{|A_2A_3|}{\sin C} \tag{19}$$

$$|PD| = \frac{|A_3A_4|}{\sin D} \tag{20}$$

If the inequalities (13), (14), (15) and (16) are respectively considered together with the inequalities (17), (18), (19) and (20), then

$$\begin{aligned}
|PB| &= \frac{|A_1A_2|}{\sin B} > \frac{h_2 \sin D}{\sin B}, \\
|PC| &= \frac{|A_2A_3|}{\sin C} > \frac{h_3 \sin A}{\sin C}, \\
|PD| &= \frac{|A_3A_4|}{\sin D} > \frac{h_4 \sin B}{\sin D}, \\
|PA| &= \frac{|A_1A_4|}{\sin A} > \frac{h_1 \sin C}{\sin A}.
\end{aligned}$$

Therefore, it is easy to see that

$$\begin{aligned}
|PA| + |PB| + |PC| + |PD| &> \frac{h_1 \sin C}{\sin A} + \frac{h_2 \sin D}{\sin B} + \frac{h_3 \sin A}{\sin C} + \frac{h_4 \sin B}{\sin D} \\
&> 4\sqrt[4]{\frac{h_1 \sin C}{\sin A} \cdot \frac{h_2 \sin D}{\sin B} \cdot \frac{h_3 \sin A}{\sin C} \cdot \frac{h_4 \sin B}{\sin D}} \\
&= 4\sqrt[4]{|PA_1| \cdot |PA_2| \cdot |PA_3| \cdot |PA_4|}
\end{aligned}$$

by Arithmetic-Geometric Mean Inequality. □

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