A Simple Construction of the Golden Ratio

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Abstract. We construct the golden ratio by using an area bisector of a trapezoid.

Consider a trapezoid $PQRS$ with bases $PQ = b$, $RS = a$, $a < b$. Assume, in Figure 1, that the segment $MN$ of length $\sqrt{\frac{a^2 + b^2}{2}}$ is parallel to $PQ$. Then $MN$ lies between the bases $PQ$ and $RS$ (see [1, p.57]). It is easy to show that $MN$ bisects the area of the trapezoid. It is more interesting to note that $M$ and $N$ divide $SP$ and $RQ$ in the golden ratio if $b = 3a$. To see this, construct a segment $SW$ parallel to $RQ$ and let $V = MN \cap SW$. It is clear that

$$\frac{SM}{SP} = \frac{MV}{PW} = \frac{\sqrt{\frac{a^2 + b^2}{2}} - a}{b - a} = \frac{\sqrt{5} - 1}{2}$$

if $b = 3a$.

Based upon this result, we present the following simple division of a given segment $AB$ in the golden ratio. Construct

1. a trapezoid $ABCD$ with $AD \parallel BC$ and $BC = 3 \cdot AD$,
2. a right triangle $BCD$ with a right angle at $C$ and $CE = AD$,
3. the midpoint $F$ of $BE$ and a point $H$ on the perpendicular bisector of $BE$ such that $FH = \frac{1}{2}BE$,
4. a point $I$ on $BC$ such that $BI = BH$.

Complete a parallelogram $BIJG$ with $J$ on $DC$ and $G$ on $AB$. See Figure 2. Then $G$ divides $AB$ in the golden ratio, i.e., $\frac{AB}{AG} = \frac{\sqrt{5} - 1}{2}$.

Proof. The trapezoid $ABCD$ has $AD = a$, $BC = b$ with $b = 3a$. The segment $JG$ is parallel to the bases and

$$JG = BI = BH = \sqrt{2} \cdot \frac{\sqrt{a^2 + b^2}}{2} = \sqrt{\frac{a^2 + b^2}{2}}.$$

Therefore, $\frac{AG}{AB} = \frac{\sqrt{5} - 1}{2}$. □

Reference


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