

## Another 5-step Division of a Segment in the Golden Section

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**Abstract.** We give one more 5-step division of a segment into golden section, using ruler and compass only.

Inasmuch as we have given in [1, 2] 5-step constructions of the golden section we present here another very simple method using ruler and compass only. It is fascinating to discover how simple the golden section appears. For two points  $P$  and  $Q$ , we denote by  $P(Q)$  the circle with  $P$  as center and  $PQ$  as radius.

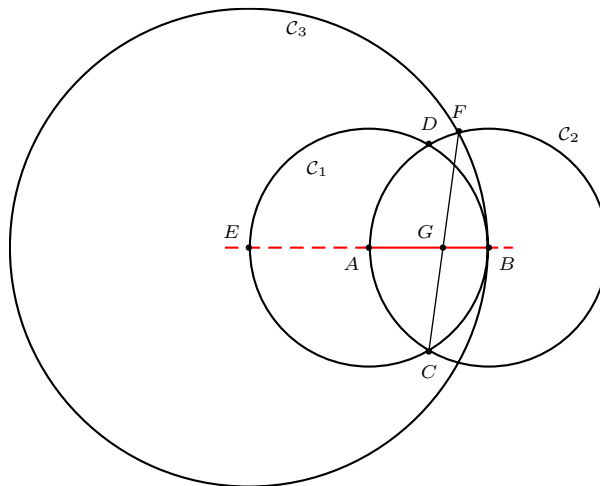


Figure 1

**Construction.** Given a segment  $AB$ , construct

- (1)  $C_1 = A(B)$ ,
- (2)  $C_2 = B(A)$ , intersecting  $C_1$  at  $C$  and  $D$ ,
- (3) the line  $AB$  to intersect  $C_1$  at  $E$  (apart from  $B$ ),
- (4)  $C_3 = E(B)$  to intersect  $C_2$  at  $F$  (so that  $C$  and  $F$  are on opposite sides of  $AB$ ),
- (5) the segment  $CF$  to intersect  $AB$  at  $G$ .

The point  $G$  divides the segment  $AB$  in the golden section.

*Proof.* Suppose  $AB$  has unit length. It is enough to show that  $AG = \frac{1}{2}(\sqrt{5} - 1)$ .

Extend  $BA$  to intersect  $C_3$  at  $H$ . Let  $CD$  intersect  $AB$  at  $I$ , and let  $J$  be the orthogonal projection of  $F$  on  $AB$ . In the right triangle  $HFB$ ,  $BH = 4$ ,  $BF = 1$ . Since  $BF^2 = BJ \times BH$ ,  $BJ = \frac{1}{4}$ . Therefore,  $IJ = \frac{1}{4}$ . It also follows that  $JF = \frac{1}{4}\sqrt{15}$ .

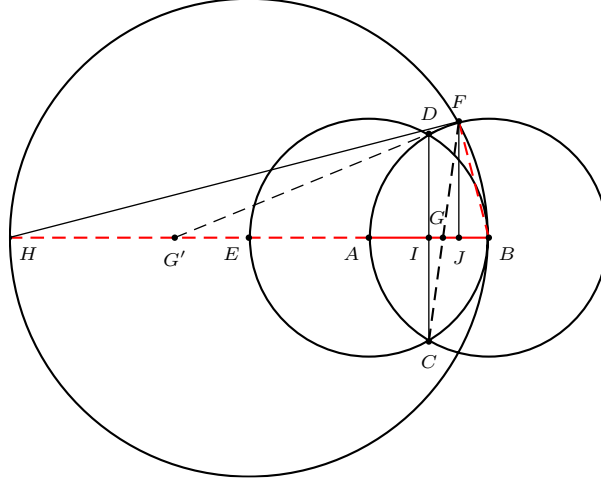


Figure 2

Now,  $\frac{IG}{GJ} = \frac{IC}{JF} = \frac{\frac{1}{2}\sqrt{3}}{\frac{1}{4}\sqrt{15}} = \frac{2}{\sqrt{5}}$ . It follows that  $IG = \frac{2}{\sqrt{5}+2} \cdot IJ = \frac{\sqrt{5}-2}{2}$ , and  $AG = \frac{1}{2} + IG = \frac{\sqrt{5}-1}{2}$ . This shows that  $G$  divides  $AB$  in the golden section.  $\square$

*Remark.* If  $FD$  is extended to intersect  $AH$  at  $G'$ , then  $G'$  is such that  $G'A : AB = \frac{1}{2}(\sqrt{5} + 1) : 1$ .

After the publication of [2], Dick Klingens and Marcello Tarquini have kindly written to point out that the same construction had appeared in [3, p.51] and [4, S.37] almost one century ago.

## References

- [1] K. Hofstetter, A simple construction of the golden section, *Forum Geom.*, 2 (2002) 65–66.
- [2] K. Hofstetter, A 5-step division of a segment in the golden section, *Forum Geom.*, 3 (2003) 205–206.
- [3] E. Lemoine, *Géométrie ou Art des Constructions Géométriques*, C. Naud, Paris, 1902.
- [4] J. Reusch, *Planimetrische Konstruktionen in Geometrischer Ausführung*, Teubner, Leipzig, 1904.

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