Forum Geometricorum
Volume 5 (2005) 149–164.

FORUM GEOM
ISSN 1534-1178

On the Complement of the Schiffler Point

Khoa Lu Nguyen

Abstract. Consider a triangle ABC with excircles (I_a), (I_b), (I_c), tangent to the nine-point circle respectively at F_a, F_b, F_c. Consider also the polars of A, B, C with respect to the corresponding excircles, bounding a triangle XYZ. We present, among other results, synthetic proofs of (i) the perspectivity of XYZ and $F_aF_bF_c$ at the complement of the Schiffler point of ABC, (ii) the concurrency at the same point of the radical axes of the nine-point circles of triangles I_aBC, I_bCA, and I_cAB.

1. Introduction

Consider a triangle ABC with excircles (I_a), (I_b), (I_c). It is well known that the nine-point circle (W) is tangent externally to the each of the excircles. Denote by F_a, F_b, and F_c the points of tangency. Consider also the polars of the vertices A with respect to (I_a), B with respect to (I_b), and C with respect to (I_c). These are the lines B_aC_a, C_bA_b, and A_cB_c joining the points of tangency of the excircles with the sidelines of triangle ABC. Let these polars bound a triangle XYZ. See Figure 1. Juan Carlos Salazar [12] has given the following interesting theorem.

Theorem 1 (Salazar). *The triangles XYZ and $F_aF_bF_c$ are perspective at a point on the Euler line.*

Darij Grinberg [3] has identified the perspector as the triangle center X_{442} of [6], the complement of the Schiffler point. Recall that the Schiffler point S is the common point of the Euler lines of the four triangles IBC, ICA, IAB, and ABC, where I is the incenter of ABC. Denote by A', B', C' the midpoints of the sides BC, CA, AB respectively, so that $A'B'C'$ is the medial triangle of ABC, with incenter I' which is the complement of I. Grinberg suggested that the lines XF_a, YF_b and ZF_c are the Euler lines of triangles $I'B'C'$, $I'C'A'$ and $I'A'B'$ respectively. The present author, in [10], conjectured the following result.

Theorem 2. *The radical center of the nine-point circles of triangles I_aBC, I_bCA and I_cAB is a point on the Euler line of triangle ABC.*

Subsequently, Jean-Pierre Ehrmann [1] and Paul Yiu [13] pointed out that this radical center is the same point S', the complement of the Schiffler point S. In this paper, we present synthetic proofs of these results, along with a few more interesting results.

Publication Date: October 18, 2005. Communicating Editor: Paul Yiu.

The author is extremely grateful to Professor Paul Yiu for his helps in the preparation of this paper.

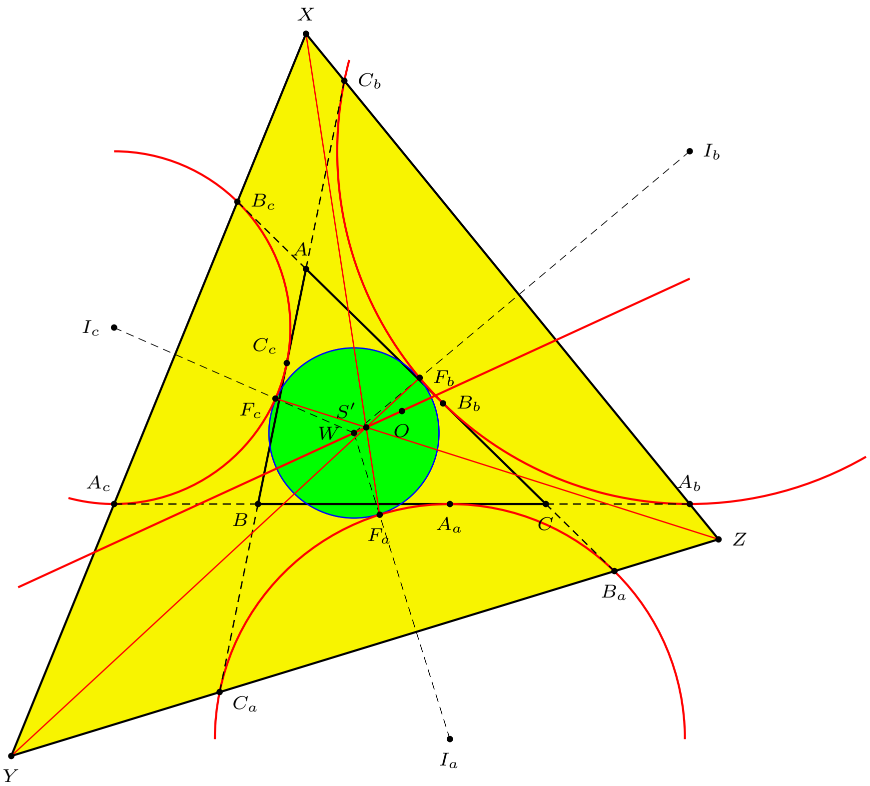

Figure 1.

2. Notations

a, b, c	Lengths of sides BC, CA, AB
R, r, s	Circumradius, inradius, semiperimeter
r_a, r_b, r_c	Exradii
O, G, W, H,	Circumcenter, centroid, nine-point center, orthocenter
I, F, S, M	Incenter, Feuerbach point, Schiffler point, Mittenpunkt
P'	Complement of P in triangle ABC
A', B', C'	Midpoints of BC, CA, AB
A_1, B_1, C_1	Points of tangency of incircle with BC, CA, AB
I_a, I_b, I_c	Excenters
F_a, F_b, F_c	Points of tangency of the nine-point circle with the excircles
A_a, B_a, C_a	Points of tangency of the A-excircle with the lines BC, CA, AB; similarly for A_b, B_b, C_b and A_c, B_c, C_c
W_a, W_b, W_c	Nine-point centers of I_aBC, I_bCA, I_cAB
M_a, M_b, M_c	Midpoints of AI_a, BI_b, CI_c
X	$A_bC_b \cap A_cB_c$; similarly for Y, Z
X_b, X_c	Orthogonal projections of B on CI_a and C on BI_a; similarly for Y_c, Y_a, Z_a, Z_b
J_a	Midpoint of arc BC of circumcircle not containing A; similarly for J_b, J_c
K_a	$A_bF_b \cap A_cF_c$; similarly for K_b, K_c

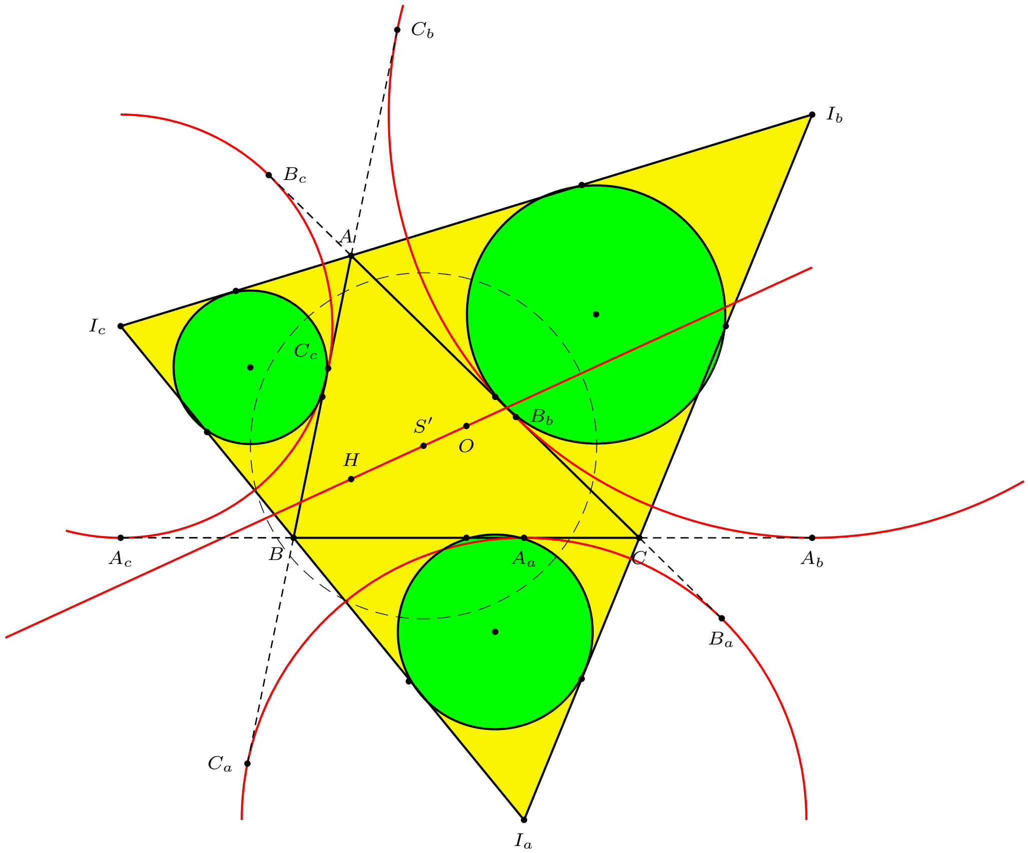

Figure 2.

3. Some preliminary results

We shall make use of the notion of directed angle between two lines. Given two lines a and b, the directed angle $(\mathsf{a},\ \mathsf{b})$ is the angle of counterclockwise rotation from a to b. It is defined modulo $180°$. We shall make use of the following basic properties of directed angles. For further properties of directed angles, see [7].

Lemma 3. (i) *For arbitrary lines* a, b, c,

$$(\mathsf{a},\ \mathsf{b}) + (\mathsf{b},\ \mathsf{c}) \equiv (\mathsf{a},\ \mathsf{c}) \bmod 180°.$$

(ii) *Four points* A, B, C, D *are concyclic if and only if* $(AC, CB) = (AD, DB)$.

Lemma 4. *Let* (O) *be a circle tangent externally to two circles* (O_a) *and* (O_b) *respectively at* A *and* B. *If* PQ *is a common external tangent of* (O_a) *and* (O_b), *then the quadrilateral* $APQB$ *is cyclic, and the lines* AP, BQ *intersect on the circle* (O).

Proof. Let PA intersect (O) at K. Since (O) and (O_a) touch each other externally at A, OK is parallel to O_aP. On the other hand, O_aP is also parallel to O_bQ as they are both perpendicular to the common tangent PQ. Therefore KO is parallel

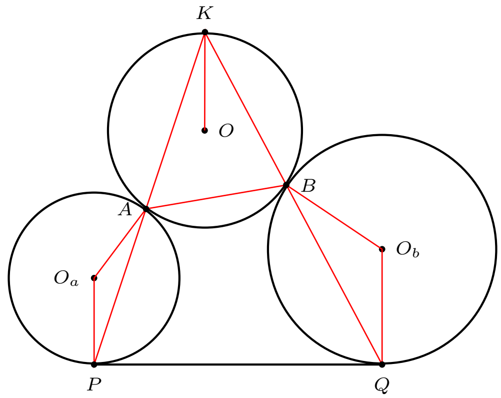

Figure 3

to O_bQ in the same direction. This implies that K, B, Q are collinear since (O_b) and (O) touch each other at externally at B. Therefore

$$(PQ, QB) = \frac{1}{2}(QO_b, O_bB) = \frac{1}{2}(KO, OB) = (KA, AB) = (PA, AB),$$

and $APQB$ is cyclic. □

We shall make use of the following results.

Lemma 5. *Let ABC be a triangle inscribed in a circle (O), and points M and N lying on AB and AC respectively. The quadrilateral $BNMC$ is cyclic if and only if MN is perpendicular to OA.*

Theorem 6. *The nine-point circles of ABC, I_aBC, I_aCA, and I_aAB intersect at the point F_a.*

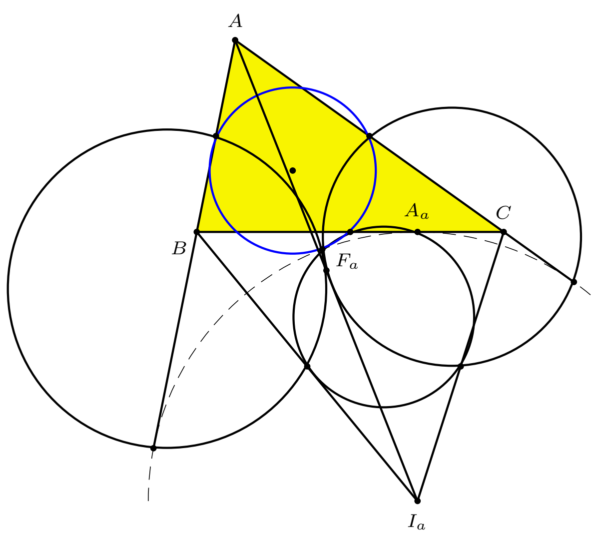

Figure 4.

Proposition 7. *The circle with diameter A_aM_a contains the point F_a.*

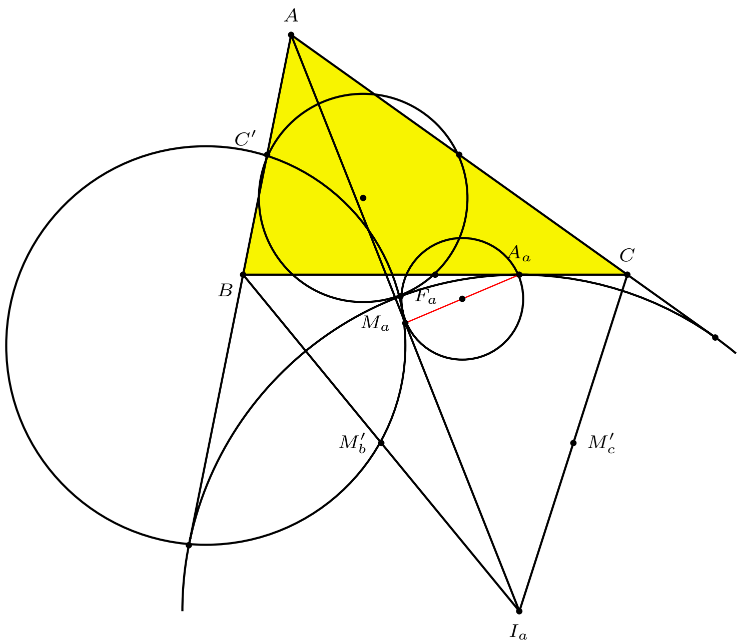

Figure 5.

Proof. Denote by M'_b and M'_c the midpoints of I_aB and I_aC respectively. The point F_a is common to the nine-point circles of I_aBC, I_aCA and I_aAB. See Figure 5. We show that $(A_aF_a, F_aM_a) = 90°$.

$$\begin{aligned}(A_aF_a, F_aM_a) =&(A_aF_a, F_aM'_b) + (M'_bF_a, F_aM_a)\\ =&(A_aM'_c, M'_cM_b) + (M'_bC', C'M_a)\\ =&-(I_aM'_c, M'_cM'_b) - (BI_a, I_aA)\\ =&-((I_aC, BC) + (BI_a, I_aA)) = 90°.\end{aligned}$$

□

4. Some properties of triangle XYZ

In this section we present some important properties of the triangle XYZ.

4.1. *Homothety with the excentral triangle.* Since YZ and I_bI_c are both perpendicular to the bisector of angle A, they are parallel. Similarly, ZX and XY are parallel to I_cI_a and I_aI_b respectively. The triangle XYZ is therefore homothetic to the excentral triangle $I_aI_bI_c$. See Figure 7. We shall determine the homothetic center in Theorem 11 below.

4.2. *Perspectivity with ABC.* Consider the orthogonal projections P and P' of A and X on the line BC. We have

$$A_cP : PA_b = (s-c) + c\cos B : (s-b) + b\cos C = s-b : s-c$$

by a straightforward calculation.

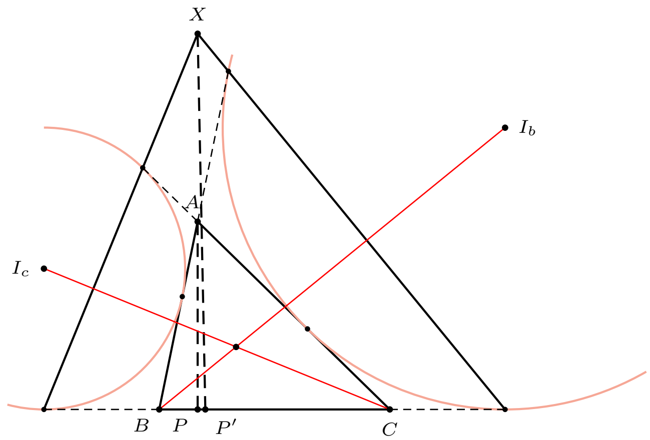

Figure 6.

On the other hand,

$$\begin{aligned} A_cP' : P'A_b =& \cot XA_cA_b : \cot XA_bA_c \\ =& \cot\left(90^\circ - \frac{C}{2}\right) : \cot\left(90^\circ - \frac{B}{2}\right) \\ =& \tan\frac{C}{2} : \tan\frac{B}{2} \\ =& \frac{1}{s-c} : \frac{1}{s-b} \\ =& s-b : s-c. \end{aligned}$$

It follows that P and P' are the same point. This shows that the line XA is perpendicular to BC and contains the orthocenter H of triangle ABC. The same is true for the lines YB and ZX. The triangles XYZ and ABC are perspective at H.

4.3. *The circumcircle of XYZ.* Applying the law of sines to triangle AXB_c, we have

$$XA = (s-b)\cdot\frac{\sin\left(90^\circ - \frac{C}{2}\right)}{\sin\frac{C}{2}} = (s-b)\cot\frac{C}{2} = r_a.$$

It follows that $HX = 2R\cos A + r_a = 2R + r$. See Figure 4. Similarly, $HY = HZ = 2R + r$. Therefore, triangle XYZ has circumcenter H and circumradius $2R + r$.

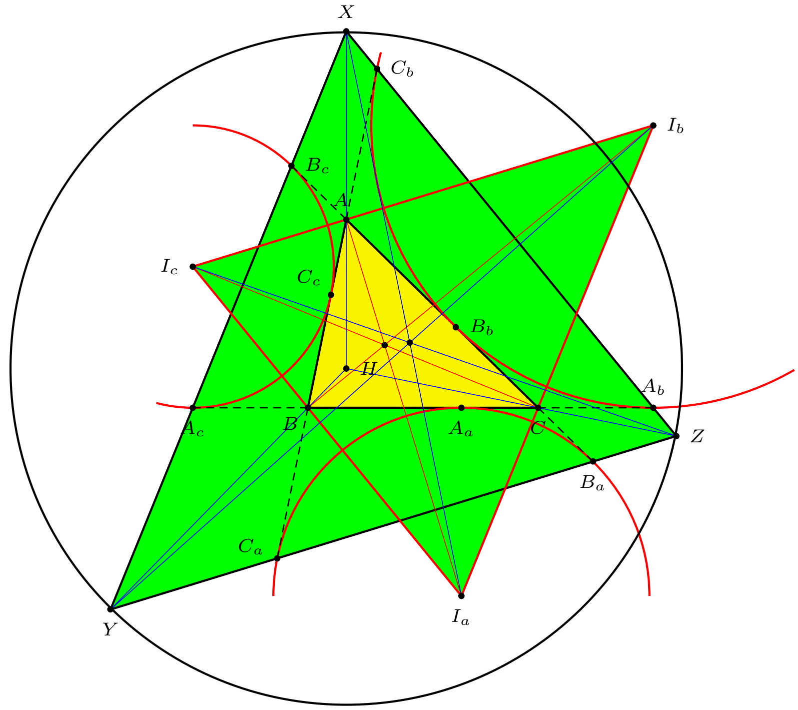

Figure 7.

5. The Taylor circle of the excentral triangle

Consider the excentral triangle $I_aI_bI_c$ with its orthic triangle ABC. The orthogonal projections Y_a and Z_a of A on I_aI_c and I_aI_b, Z_b and X_b of B on I_bI_c and I_aI_b, together with X_c and Y_c of C on I_bI_c and I_cI_a are on a circle called the Taylor circle of the excentral triangle. See Figure 8.

Proposition 8. *The points X_b, X_c lie on the line YZ.*

Proof. The collinearity of C_a, X_b, X_c follows from

$$\begin{aligned}(C_aX_b, X_bB) =&(C_aI_a, I_aB)\\ =&(C_aI_a, AB) + (AB, I_aB)\\ =&90^\circ + (I_aB, BC)\\ =&(X_cC, I_aB) + (I_aB, BC)\\ =&(X_cC, CB)\\ =&(X_cX_b, X_bB).\end{aligned}$$

Similarly, X_b is also on the line YZ, and Z_a, Z_b are on the line XY, Y_c, Y_a are on the line XZ. □

Proposition 9. *The line Y_aZ_a contains the midpoints B', C' of CA, AB, and is parallel to BC.*

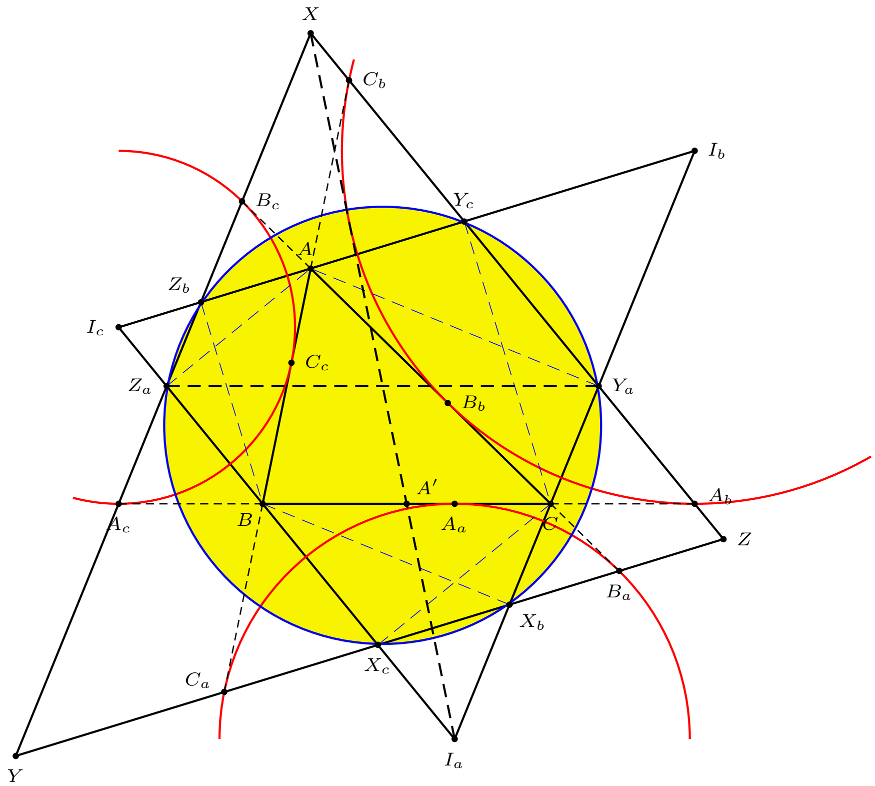

Figure 8.

Proof. Since A, Y_a, I_a, Z_a are concyclic,

$$(AY_a, Y_aZ_a) = (AI_a, I_aZ_a) = \frac{C}{2} = (CA, AY_a).$$

Therefore, the intersection of AC and Y_aZ_a is the circumcenter of the right triangle ACY_a, and is the midpoint B' of CA. Similarly, the intersection of AB and Y_aZ_a is the midpoint C' of AB. □

Proposition 10. *The line I_aX contains the midpoint A' of BC.*

Proof. Since the diagonals of the parallelogram $I_aY_aXZ_a$ bisect each other, the line I_aX passes through the midpoint of the segment Y_aZ_a. Since Y_aZ_a and BC are parallel, with B on I_aZ_a and C on I_aY_a, the same line I_aX also passes through the midpoint of the segment BC. □

Theorem 11. *The triangles XYZ and $I_aI_bI_c$ are homothetic at the Mittenpunkt M of triangle ABC, the ratio of homothety being $2R + r : -2R$.*

Proof. The lines I_aX, I_bY, I_cZ contain respectively the midpoints of A', B', C' of BC, CA, AB. They intersect at the common point of I_aA', I_bB', I_cC', the Mittenpunkt M of triangle ABC. This is the homothetic center of the triangles XYZ and $I_aI_bI_c$. The ratio of homothety of the two triangle is the same as the ratio of their circumradii. □

Theorem 12. *The Taylor circle of the excentral triangle is the radical circle of the excircles.*

Proof. The perpendicular bisector of Y_cZ_b is a line parallel to the bisector of angle A and passing through the midpoint A' of BC. This is the A'-bisector of the medial triangle $A'B'C'$. Similarly, the perpendicular bisectors of Z_aX_c and X_bY_a are the other two angle bisectors of the medial triangle. These three intersect at the incenter of the medial triangle, the Spieker center of ABC.

It is well known that S_{p} is also the center of the radical circle of the excircles. To show that the Taylor circle coincides with the radical circle, we show that they have equal radii. This follows easily from

$$I_aX_c \cdot I_aZ_a = \frac{r_a \sin\frac{A}{2}}{\cos\frac{C}{2}} \cdot I_aA\cos\frac{C}{2} = r_a \cdot I_aA\sin\frac{A}{2} = r_a^2.$$

□

6. Proofs of Theorems 1 and 2

We give a combined proof of the two theorems, by showing that the line XF_a is the radical axis of the nine-point circles (W_b) and (W_c) of triangles I_bCA and I_cAB. In fact, we shall identify some interesting points on this line to show that it is also the Euler line of triangle $I'B'C'$.

6.1. *XF_a as the radical axis of (W_b) and (W_c).*

Proposition 13. *X lies on the radical axis of the circles (W_b) and (W_c).*

Proof. By Theorem 12, $XZ_a \cdot XZ_b = XY_a \cdot XY_c$. Since Y_c, Y_a are on the nine-point circle (W_b) and Z_a, Z_b on the the circle (W_c), X lies on the radical axis of these two nine-point circles. □

Since AZ_a and AY_a are perpendicular to I_aI_c and I_aI_b, and $I_aI_bI_c$ and XYZ are homothetic, A is the orthocenter of triangle XY_aZ_a. It follows that X is the orthocenter of AY_aZ_a. Since $(AY_a, Y_aI_a) = (AZ_a, Z_aI_a) = 90°$, the triangle AY_aZ_a has circumcenter the midpoint M_a of AI_a. It follows that XM_a is the Euler line of triangle AY_aZ_a.

Proposition 14. *M_a lies on the radical axis of the circles (W_b) and (W_c).*

Proof. Let M_b'' and M_c'' be the midpoints of AI_b and AI_c respectively. See Figure 9. Note that these lie on the nine-point circles (W_b) and (W_c) respectively. Since C, I_b, I_c, B are concyclic, we have $I_aB \cdot I_aI_c = I_aC \cdot I_aI_b$. Applying the homthety $\mathsf{h}(A, \frac{1}{2})$, we have the collinearity of M_a, C', M_c'', and of M_a, B', M_b'', Furthermore, $M_aC' \cdot M_aM_c'' = M_aB' \cdot M_aM_b''$. This shows that M_a lies on the radical axis of (W_b) and (W_c). □

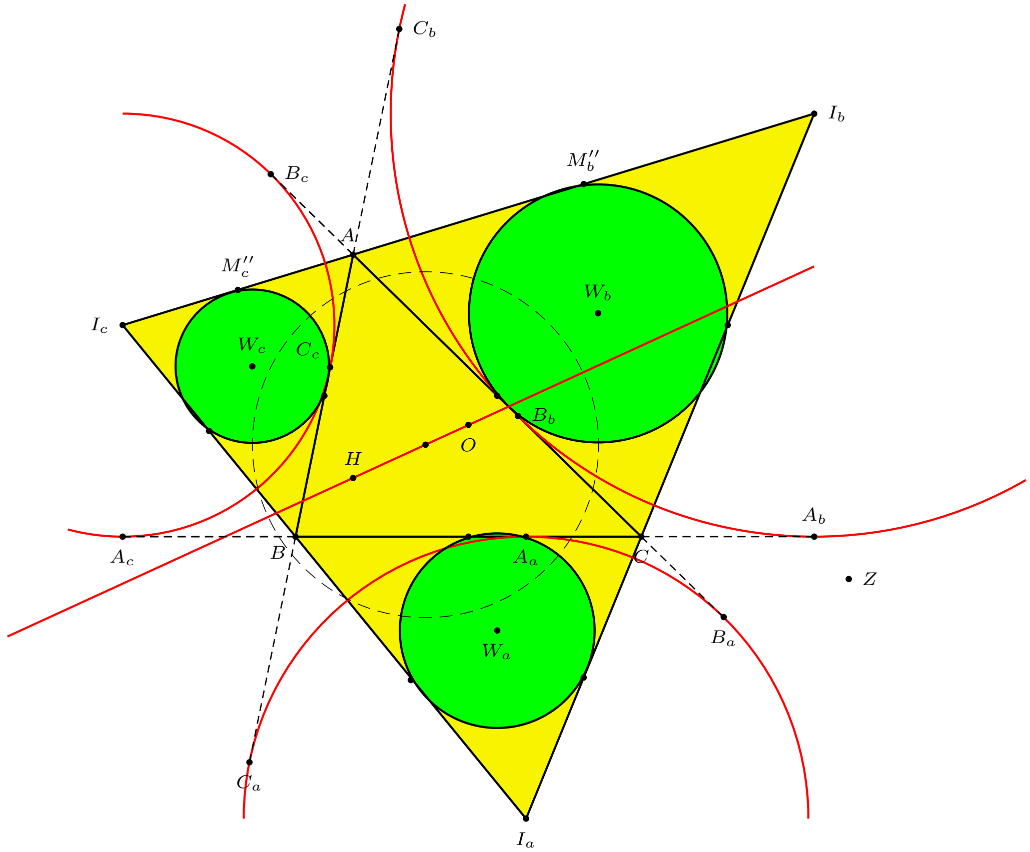

Figure 9.

Proposition 15. *X, F_a, and M_a are collinear.*

Proof. We prove that the Euler line of triangle AY_aZ_a contains the point F_a. The points X and M_a are respectively the orthocenter and circumcenter of the triangle.

Let A'_a be the antipode of A_a on the A-excircle. Since AX has length r_a and is perpendicular to BC, XAA'_aI_a is a parallelogram. Therefore, XA'_a contains the midpoint M_a of AI_a.

By Proposition 7, $(A_aF_a, F_aM_a) = 90°$. Clearly, $(A_aF_a, F_aA'_a) = 90°$. This means that F_a, M_a, and A'_a are collinear. The line containing them also contains X. □

Proposition 16. *XF_a is also the Euler line of triangle AY_aZ_a.*

Proof. The circumcenter of AY_aZ_a is clearly M_a. On the other hand, since A is the orthocenter of triangle XY_aZ_a, X is the orthocenter of triangle AY_aZ_a. Therefore the line XM_a, which also contains F_a, is the Euler line of triangle AY_aZ_a. □

6.2. *XF_a as the Euler line of triangle $I'B'C'$.*

Proposition 17. *M_a is the orthocenter of triangle $I'B'C'$.*

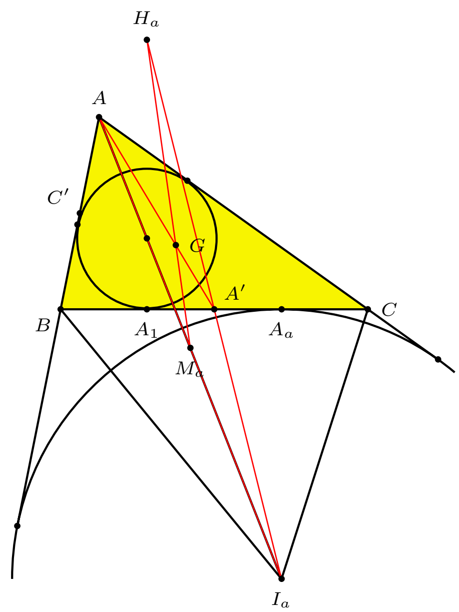

Figure 10.

Proof. Let H_a be the orthocenter of IBC. Since BH_a is perpendicular to IC, it is parallel to I_aC. Similarly, CH_a is parallel to I_aB. Thus, BH_aCI_a is a parallelogram, and A' is the midpoint of I_aH_a. Consider triangle AI_aH_a which has M_a and A' for the midpoints of two sides. The intersection of M_aH_a and AA' is the centroid of the triangle, which coincides with G. Furthermore,

$$GH_a : GM_a = GA : GA' = 2 : -1.$$

Hence, M_a is the orthocenter of $I'B'C'$. □

Proposition 18. *K_a is the circumcenter of $I'B'C'$.*

Proof. By Lemma 4, the points F_b, F_c, A_b and A_c are concyclic, and the lines A_bF_b and A_cF_c intersect at a point K_a on the nine-point circle, which is the midpoint of the arc $B'C'$ not containing A'. See Figure 11. The image of K_a under $\mathsf{h}(G, -2)$ is J_a, the circumcenter of IBC. It follows that K_a is the circumcenter of $I'B'C'$. □

Proposition 19. *K_a lies on the radical axis of (W_b) and (W_c).*

Proof. Let D and E be the second intersections of K_aF_b with (W_b) and K_aF_c with (W_c) respectively. We shall show that $K_aF_b \cdot K_aD = K_aF_c \cdot K_aE$.

Since A_c, F_c, F_b, A_b are concyclic, we have $K_aF_c \cdot K_aA_c = K_aF_b \cdot K_aA_b = k$, say. Note that

$$A_cE \cdot A_cF_c = A_cZ_a \cdot A_cZ_b = \frac{(s-a)^2 \sin\left(B + \frac{A}{2}\right)}{\tan\frac{B}{2}\cos\frac{A}{2}}.$$

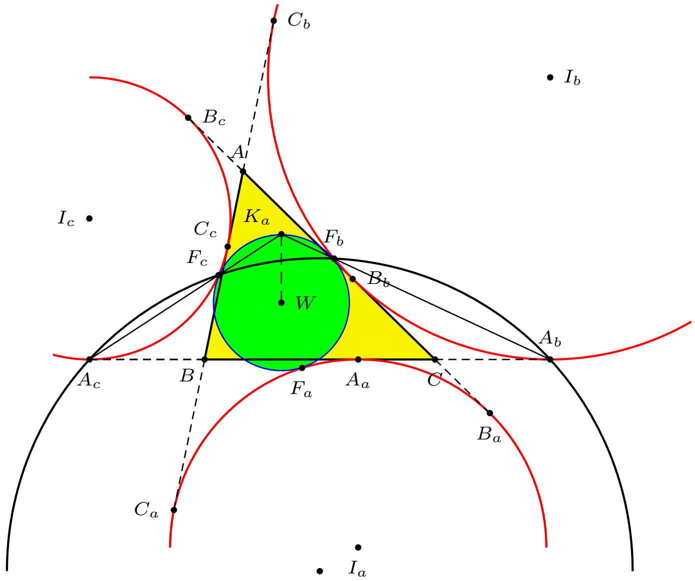

Figure 11.

Since (I_c) and (W) extouch at F_c, we have $\frac{K_aF_c}{A_cF_c} = -\frac{R}{2r_a}$. Therefore,

$$\begin{aligned}\frac{A_cE}{K_aA_c} &= \frac{K_aF_c}{A_cF_c} \cdot \frac{A_cE \cdot A_cF_c}{K_aF_c \cdot K_aA_c} \\ &= -\frac{R}{2r_a} \cdot \frac{(s-a)^2 \sin\left(B + \frac{A}{2}\right)}{k \cdot \tan\frac{B}{2} \cos\frac{A}{2}} \\ &= -\frac{R(s-a)^2 \sin\left(B + \frac{A}{2}\right)}{k \cdot s \tan\frac{B}{2} \tan\frac{C}{2} \cos\frac{A}{2}}.\end{aligned}$$

Similarly,

$$\frac{A_bD}{K_aA_b} = -\frac{R(s-a)^2 \sin\left(C + \frac{A}{2}\right)}{k \cdot s \tan\frac{B}{2} \tan\frac{C}{2} \cos\frac{A}{2}}.$$

Since $\sin\left(B + \frac{A}{2}\right) = \sin\left(C + \frac{A}{2}\right)$, it follows that $\frac{A_bD}{K_aA_b} = \frac{A_cE}{K_aA_c}$. Hence, DE is parallel to A_bA_c. From $K_aF_b \cdot K_aA_b = K_aF_c \cdot K_aA_c$, we have $K_aF_b \cdot K_aD = K_aF_c \cdot K_aE$. This shows that K_a lies on the radical axis of (W_b) and (W_c). □

Corollary 20. *K_a lies on the line XF_a.*

6.3. *Proof of Theorems 1 and 2.* We have shown that the line XF_a is the radical axis of (W_b) and (W_c). Likewise, YF_b is that of (W_c), (W_a), and ZF_c that of (W_a), (W_b). It follows that the three lines are concurrent at the radical center of the three circles. This proves Theorem 1.

We have also shown that the line XF_a is the image of the Euler line of IBC under the homothety $\mathsf{h}(G, -\frac{1}{2})$; similarly for the lines YF_b and ZF_c. Since the Euler lines of IBC, ICA, and IAB intersect at the Schiffler point S on the Euler line of ABC, the lines XF_a, YF_b, ZF_c intersect at the complement of the Schiffler point S, also on the same Euler line. This proves Theorem 2.

7. Some further results

Theorem 21. *The six points Y, Z, A_b, A_c, F_b, F_c are concyclic.*

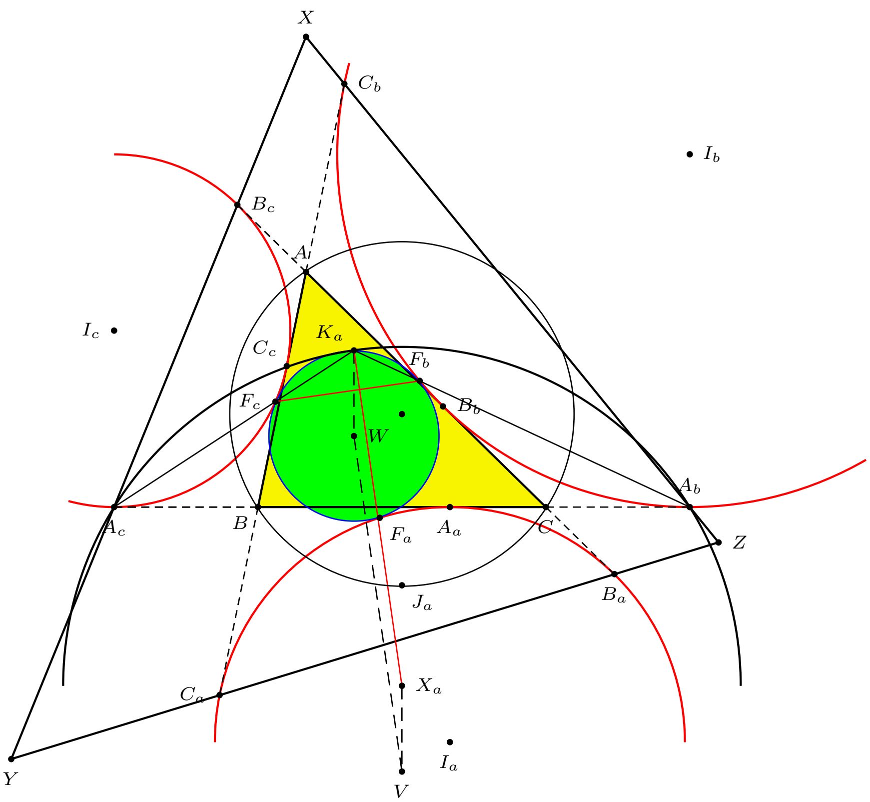

Figure 12.

Proof. (i) The points A_b, A_c, F_b, F_c are concyclic and the lines A_bF_b, A_cF_c meet at K_a. Let X_a be the circumcenter of $K_aA_bA_c$. Since F_b and F_c are points on K_aA_b and K_aA_c, and $F_bA_bA_cF_c$ is cyclic, it follows from Lemma 5 that K_aX_a is perpendicular to F_bF_c. Hence X_a is the intersection of the perpendicular from K_a to F_bF_c and the perpendicular bisector of BC. Since triangle $K_aA_bA_c$ is similar to $K_aF_cF_b$, and $A_bA_c = b + c$, its circumradius is

$$\frac{b+c}{F_bF_c} \cdot \frac{R}{2} = \frac{1}{2}\sqrt{(R + 2r_b)(R + 2r_c)}.$$

Here, we have made use of the formula

$$F_bF_c = \frac{b+c}{\sqrt{(R+2r_b)(R+2r_c)}} \cdot R$$

from [2].

(ii) A simple angle calculation shows that the points Y, Z, A_b, A_c are also concyclic. Its center is the intersection of the perpendicular bisectors of A_bA_c and YZ. The perpendicular bisector of A_bA_c is clearly the same as that of BC. Since YZ is parallel to I_bI_c, its perpendicular is the parallel through H (the circumcenter of XYZ) to the bisector of angle A.

(iii) Therefore, if this circumcenter is V, then $J_aV = AH = 2R\cos A$.

(iv) To show that the two circle $F_bA_bA_cF_c$ is the same as the circle in (ii), it is enough to show that V lies on the perpendicular bisector of F_bF_c. This is equivalent to showing that VW is perpendicular to F_bF_c. To prove this, we show that K_aWVX_a is a parallelogram. Applying the Pythagorean theorem to triangle $A'A_bX_a$, we have

$$\begin{aligned}
4A'X_a^2 =&(R+2r_b)(R+2r_c) - (b+c)^2\\
=&R^2 + 4R(r_b+r_c) + 4r_br_c - (b+c)^2\\
=&R^2 + 4R\cdot R(1+\cos A) + 4s(s-a) - (b+c)^2\\
=&R^2(1+4(1+\cos A)) - a^2\\
=&R^2(1+4(1+\cos A) - 4\sin^2 A)\\
=&R^2(1+2\cos A)^2.
\end{aligned}$$

This means that $A'X_a = \frac{R}{2}(1+2\cos A)$, and it follows that

$$\begin{aligned}
X_aV =&A'V - A'X_a = A'J + JV - A'X_a\\
=&R(1-\cos A) + 2R\cos A - \frac{R}{2}(1+2\cos A)\\
=&\frac{R}{2} = K_aW.
\end{aligned}$$

Therefore, VW, being parallel to K_aX_a, is perpendicular to F_bF_c. □

Denote by $\mathcal{C}_a$ the circle through these 6 points. Similarly define $\mathcal{C}_b$ and $\mathcal{C}_c$.

Corollary 22. *The radical center of the circles $\mathcal{C}_a$, $\mathcal{C}_b$, $\mathcal{C}_c$ is S'.*

Proof. The points X and F_a are common to the circles $\mathcal{C}_b$ and $\mathcal{C}_c$. The line XF_a is the radical axis of the two circles. Similarly the radical axes of the two other two pairs of circles are YF_b and ZF_c. The radical center is therefore S'. □

Proposition 23. *The line XA_a is perpendicular to YZ.*

Proof. With reference to Figure 8, note that

$$\begin{aligned} A_bY_a : A_bX =& A_bC \cdot \frac{\sin\left(C+\frac{A}{2}\right)}{\sin\frac{C}{2}} : A_bA_c \cdot \frac{\sin\frac{A+B}{2}}{\sin\frac{B+C}{2}} \\ =& A_bC : (b+c) \cdot \frac{\sin\frac{C}{2}\sin\frac{A+B}{2}}{\sin\left(C+\frac{A}{2}\right)\sin\frac{B+C}{2}} \\ =& A_bC : (b+c) \cdot \frac{\sin C}{\sin(C+A)+\sin C} \\ =& A_bC : c \\ =& A_bC : A_bA_a. \end{aligned}$$

This means that XA_a is parallel to Y_cC, which is perpendicularto I_bI_c and YZ. □

Corollary 24. *XYZ is perspective with the extouch triangle $A_aB_bC_c$, and the perspector is the orthocenter of XYZ.*

Remark. This is the triangle center X_{72} of [6].

Proposition 25. *The complement of the Schiffler point is the point S' which divides HW in the ratio*

$$HS' : S'W = 2(2R+r) : -R.$$

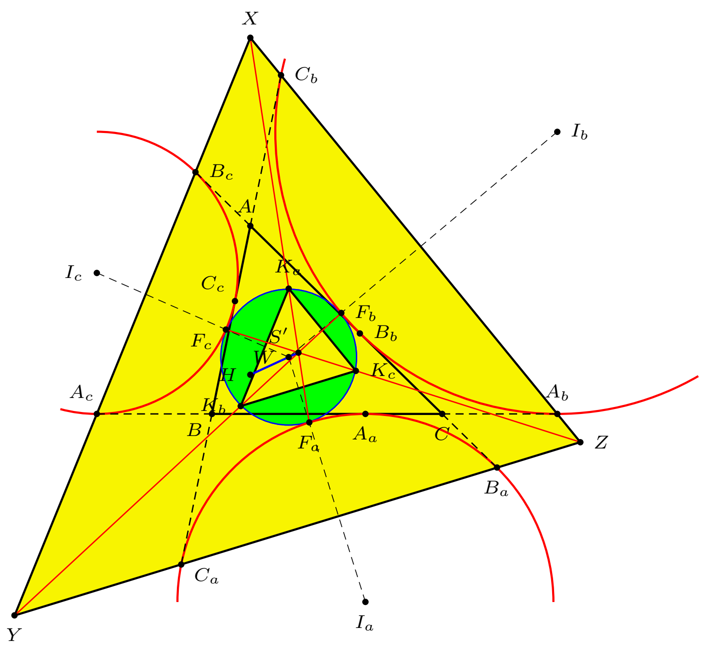

Figure 13.

Proof. We define K_b and K_c similarly as K_a. Since K_b and K_c are the midpoints of the arcs $C'A'$ and $A'B'$, K_bK_c is perpendicular to the A'-bisector of $A'B'C'$, and hence parallel to YZ. The triangle $K_aK_bK_c$ is homothetic to XYZ. The homothetic center is the common point of the lines XK_a, YK_b, and ZK_c, which are XF_a, YF_b, ZF_c. This is the complement of the Schiffler point. Since triangles $K_aK_bK_c$ and XYZ have circumcenters W, H, and circumradii $\frac{R}{2}$ and $2R+r$, this homothetic center S' divides the segment HW in the ratio given above. □

References

[1] J.-P. Ehrmann, Hyacinthos message 10564, October 1, 2004.
[2] L. Emelyanov and T. Emelyanova, A note on the Feuerbach point, *Forum Geom.*, 1 (2001) 121–124.
[3] D. Grinberg, Hyacinthos message 10342, August 31, 2004.
[4] D. Grinberg, Hyacinthos message 10562, October 1, 2004.
[5] D. Grinberg, Hyacinthos message 10587, October 3, 2004
[6] C. Kimberling, *Encyclopedia of Triangle Centers*, available at http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.
[7] R. A. Johnson, *Advanced Euclidean Geometry*, 1925, Dover reprint.
[8] K. L. Nguyen, Hyacinthos message 10384, September, 5, 2004.
[9] K. L. Nguyen, Hyacinthos message 10520, September, 23, 2004.
[10] K. L. Nguyen, Hyacinthos message 10563, October 1, 2004.
[11] K. L. Nguyen, Hyacinthos message 10913, November 28, 2004.
[12] J. C. Salazar, Hyacinthos message 10323, August 29, 2004.
[13] P. Yiu, Hyacinthos message 10565, October 1, 2004.

Khoa Lu Nguyen: 806 Candler Dr, Houston, Texas, 77037-4122, USA
E-mail address: treegoner@yahoo.com