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On the Cyclic Complex of a Cyclic Quadrilateral

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Abstract. To every cyclic quadrilateral corresponds naturally a complex of sixteen cyclic quadrilaterals. The radical axes of the various pairs of circumcircles, the various circumcenters and anticenters combine to interesting configurations. Here are studied some of these, considered to be basic in the study of the whole complex.

1. Introduction

Consider a generic convex cyclic quadrilateral $q = ABCD$. Here we consider a simple figure, resulting by constructing other quadrangles on the sides of q, similar to q. This construction was used in a recent simple proof, of the minimal area property of cyclic quadrilaterals, by Antreas Varverakis [1]. It seems though that the figure is interesting for its own. The principle is to construct the quadrilateral $q' = CDEF$, on a side of and similar to q, but with reversed orientation.

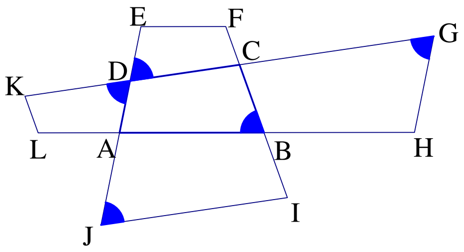

Figure 1. $CDEF$: top-flank of $ABCD$

The sides of the new align with those of the old. Besides, repeating the procedure three more times with the other sides of q, gives the previous basic Figure 1. For convenience I call the quadrilaterals: top-flank $t = CDEF$, right-flank $r = CGHB$, bottom-flank $b = BIJA$ and left-flank $l = DKLA$ of q respectively. In addition to these *main flanks*, there are some other flanks, created by the extensions of the sides of q and the extensions of sides of its four main flanks. Later create the *big flank* denoted below by q^*. To spare words, I drew in Figure 2 these sixteen quadrilaterals together with their names. All these quadrilaterals are cyclic and share the same angles with q. In general, though, only the main flanks are similar to q. More precisely, from their construction, flanks l, r are homothetic, t, b are also homothetic and two adjacent flanks, like r, t are antihomothetic with respect

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to their common vertex, here C. The symbol in braces, $\{rt\}$, will denote the circumcircle, the symbol in parentheses, (rt), will denote the circumcenter, and the symbol in brackets, $[rt]$, will denote the anticenter of the corresponding flank. Finally, a pair of symbols in parentheses, like (rt, rb), will denote the radical axis of the circumcircles of the corresponding flanks. I call this figure the *cyclic complex* associated to the cyclic quadrilateral.

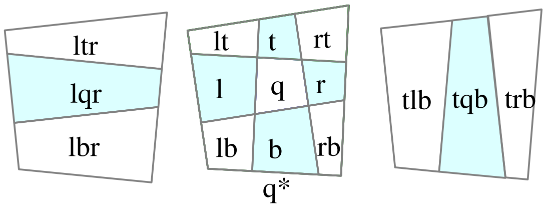

Figure 2. The cyclic complex of q

2. Radical Axes

By taking all pairs of circles, the total number of radical axes, involved, appears to be 120. Not all of them are different though. The various sides are radical axes of appropriate pairs of circles and there are lots of coincidences. For example the radical axes $(t, rt) = (q, r) = (b, rb) = (tqb, rqb)$ coincide with line BC. The same happens with every side out of the eight involved in the complex. Each side coincides with the radical axis of four pairs of circles of the complex. In order to study other identifications of radical axes we need the following:

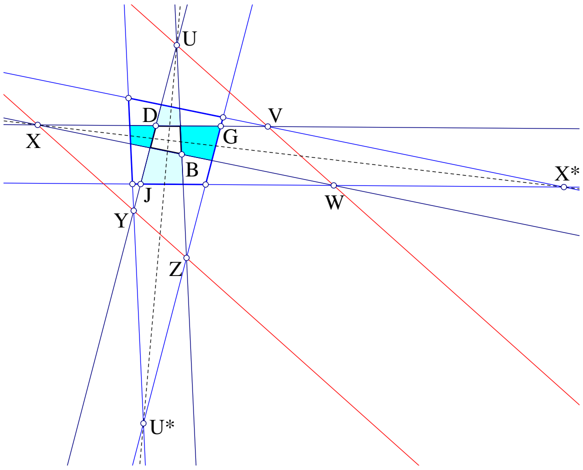

Figure 3. Intersections of lines of the complex

Lemma 1. *Referring to Figure 3, points X, U are intersections of opposite sides of q. U^*, X^* are intersections of opposite sides of $q*$. V, W, Y, Z are intersections of opposite sides of other flanks of the complex. Points X, Y, Z and U, V, W are aligned on two parallel lines.*

The proof is a trivial consequence of the similarity of opposite located main flanks. Thus, l, r are similar and their similarity center is X. Analogously t, b are similar and their similarity center is U. Besides triangles XDY, VDU are anti-homothetic with respect to D and triangles XGZ, WJU are similar to the previous two and also anti-homothetic with respect to B. This implies easily the properties of the lemma.

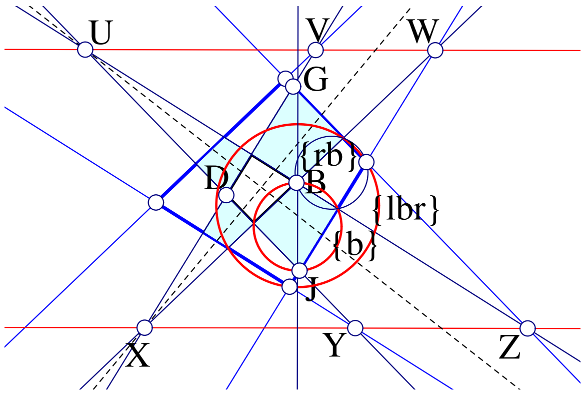

Figure 4. Other radical axes

Proposition 2. *The following lines are common radical axes of the circle pairs:*
(1) *Line XX^* coincides with $(lt, lb) = (t, b) = (rt, rb) = (ltr, lbr)$.*
(2) *Line UU^* coincides with $(lt, rt) = (l, r) = (lb, rb) = (tlb, trb)$.*
(3) *Line XYZ coincides with $(lbr, b) = (lqr, q) = (ltr, t) = (q^*, tqb)$.*
(4) *Line UVW coincides with $(tlb, l) = (tqb, q) = (trb, r) = (q^*, lqr)$.*

Referring to Figure 4, I show that line XYZ is identical with (lbr, b). Indeed, from the intersection of the two circles $\{lbr\}$ and $\{b\}$ with circle $\{rb\}$ we see that Z is on their radical axis. Similarly, from the intersection of these two circles with $\{lb\}$ we see that Y is on their radical axis. Hence line ZYX coincides with the radical axis (lbr, b). The other statements are proved analogously.

3. Centers

The centers of the cyclic complex form various parallelograms. The first of the next two figures shows the centers of the small flanks, and certain parallelograms created by them. Namely those that have sides the medial lines of the sides of the flanks. The second gives a panorama of all the sixteen centers together with a parallelogramic pattern created by them.

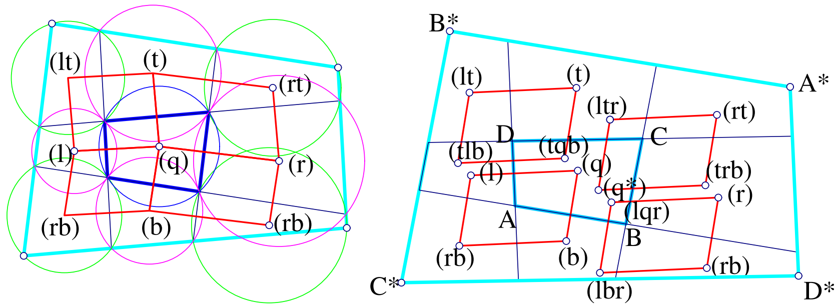

Figure 5. Panorama of centers of flanks

Proposition 3. *Referring to Figure 5, the centers of the flanks build equal parallelograms with parallel sides:* $(lt)(t)(tqb)(tlb)$, $(ltr)(rt)(trb)(q*)$, $(l)(q)(b)(rb)$ *and* $(lqr)(r)(rb)(lbr)$. *The parallelogramic pattern is symmetric with respect to the middle* M *of segment* $(q)(q*)$ *and the centers* $(tqb),(q),(q^*),(lqr)$ *are collinear.*

The proof of the various parallelities is a consequence of the coincidences of radical axes. For example, in the first figure, sides $(t)(rt)$, $(q)(r)$, $(b)(rb)$ are parallel because, all, are orthogonal to the corresponding radical axis, coinciding with line BC. In the second figure $(lt)(t),(tlb)(tqb),(l)(q),(rb)(b)$ are all orthogonal to AD. Similarly $(ltr)(rt)$, $(q*)(trb)$, $(lqr)(r)$, $(lbr)(rb)$ are orthogonal to A^*D^*. The parallelity of the other sides is proved analogously. The equality of the parallelograms results by considering other implied parallelograms, as, for example, $(rb)(b)(t)(lt)$, implying the equality of horizontal sides of the two left parallelograms. Since the labeling is arbitrary, any main flank can be considered to be the left flank of the complex, and the previous remarks imply that all parallelograms shown are equal. An importand case, in the second figure, is that of the collinearity of the centers $(tqb),(q),(q^*),(lqr)$. Both lines $(tqb)(q)$ and $(q^*)(lqr)$ are orthogonal to the axis XYZ of the previous paragraph. $(tqb)(q^*)$ is orthogonal to the axis UVW, which, after lemma-1, is parallel to XYZ, hence the collinearity. In addition, from the parallelograms, follows that the lengths are equal: $|(tqb)(q)|=|(q^*)(lqr)|$. The symmetry about M is a simple consequence of the previous considerations.

There are other interesting quadrilaterals with vertices at the centers of the flanks, related directly to q. For example the next proposition relates the centers of the main flanks to the anticenter of the original quadrilateral q. Recall that the *anticenter* is the symmetric of the circumcenter with respect to the centroid of the quadrilateral. Characteristically it is the common intersection point of the orthogonals from the middles of the sides to their opposites. Some properties of the anticenter are discussed in Honsberger [2]. See also Court [3] and the miscelanea (remark after Proposition 11) below.

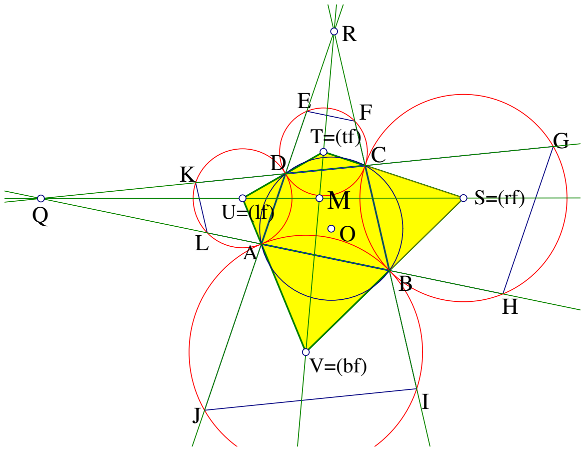

Figure 6. Anticenter through the flanks

Proposition 4. *Referring to Figure 6, the following properties are valid:*
(1) *The circumcircles of adjacent main flanks are tangent at the vertices of q.*
(2) *The intersection point of the diagonals US, TV of the quadrilateral $TSUV$, formed by the centers of the main flanks, coincides with the anticenter M of q.*
(3) *The intersection point of the diagonals of the quadrilateral formed by the anticenters of the main flanks coincides with the circumcenter O of q.*

The properties are immediate consequences of the definitions. (1) follows from the fact that triangles FTC and CSB are similar isosceli. To see that property (2) is valid, consider the parallelograms $p = OYMX$, $p^{**} = UXM^{**}Y$ and $p^* = SY^*M^*Y$, tightly related to the anticenters of q and its left and right flanks (Figure 7). X, Y, X^* and Y^* being the middles of the respective sides. One sees easily that triangles UXM and MYS are similar and points U, M, S are aligned. Thus the anticenter M of q lies on line US, passing through Q. Analogously it must lie also on the line joining the two other circumcenters. Thus, it coincides with their intersection.

The last assertion follows along the same arguments, from the similarity of parallelograms $UX^*M^{**}X$ and SYM^*Y^* of Figure 7. O is on the line M^*M^{**}, which is a diagonal of the quadrangle with vertices at the anticenters of the flanks.

Proposition 5. *Referring to the previous figure, the lines QM and QO are symmetric with respect to the bisector of angle AQD. The same is true for lines QX and QY.*

This is again obvious, since the trapezia $DAHG$ and $CBLK$ are similar and inversely oriented with respect to the sides of the angle AQD.

Remark. One could construct further flanks, left from the left and right from the right flank. Then repeat the procedure and continuing that way fill all the area of the angle AQD with flanks. All these having alternatively their anticenters and

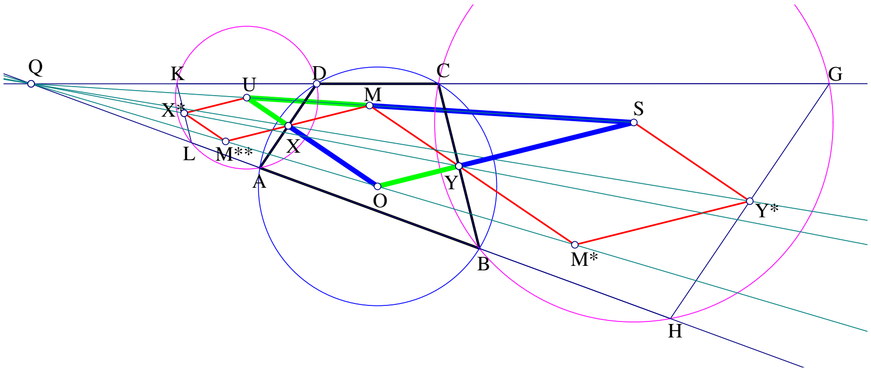

Figure 7. Anticenters M, M^*, M^{**} of the flanks

circumcenters on the two lines QO and QM and the centers of their sides on the two lines OX and OY.

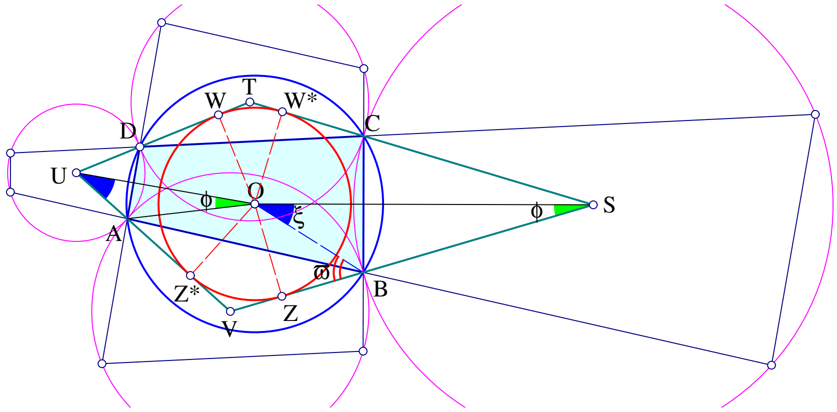

Figure 8. The circumscriptible quadrilateral $STUV$

Proposition 6. *The quadrilateral $STUV$, of the centers of the main flanks, is circumscriptible, its incenter coincides with the circumcenter O of q and its radius is $r \cdot sin(\phi + \xi)$. r being the circumcradius of q and $2\phi, 2\xi$ being the measures of two angles at O viewing two opposite sides of q.*

The proof follows immediately from the similarity of triangles UAO and OBS in Figure 8. The angle $\omega = \phi + \xi$, gives for $|OZ| = r \cdot sin(\omega)$. Z, Z^*, W, W^* being the projections of O on the sides of $STUV$. Analogous formulas hold for the other segments $|OZ^*| = |OW| = |OW^*| = r \cdot sin(\omega)$.

Remarks. (1) Referring to Figure 8, Z, Z^*, W, W^* are vertices of a cyclic quadrilateral q', whose sides are parallel to those of $ABCD$.

(2) The distances of the vertices of q and q' are equal: $|ZB| = |AZ^*| = |DW| = |CW^*| = r \cdot cos(\phi + \xi)$.

(3) Given an arbitrary circumscriptible quadrilateral $STUV$, one can construct the cyclic quadrangle $ABCD$, having centers of its flanks the vertices of $STUV$. Simply take on the sides of $STUV$ segments $|ZB| = |AZ^*| = |DW| = |CW^*|$ equal to the above measure. Then it is an easy exercise to show that the circles centered at S, U and passing from B, C and A, D respectively, define with their intersections on lines AB and CD the right and left flank of $ABCD$.

4. Anticenters

The anticenters of the cyclic complex form a parallelogramic pattern, similar to the previous one for the centers. The next figure gives a panoramic view of the sixteen anticenters (in blue), together with the centers (in red) and the centroids of flanks (white).

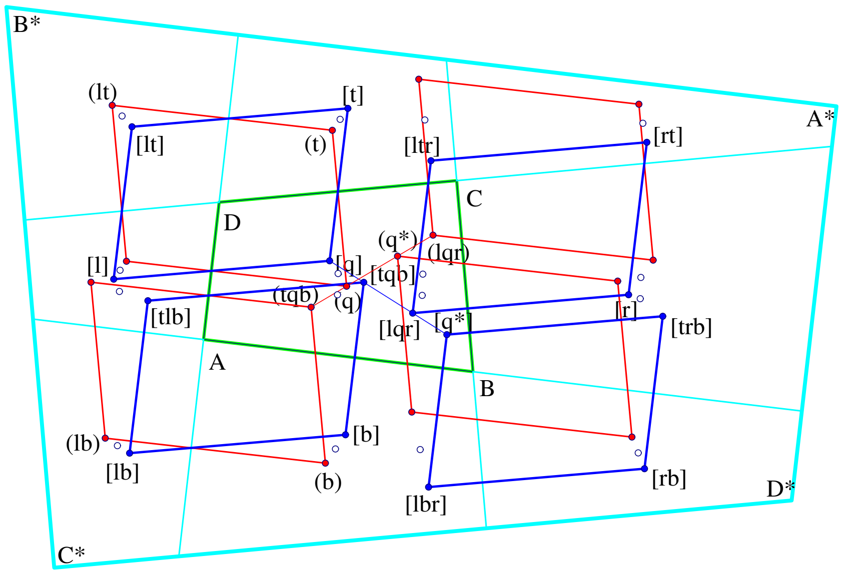

Figure 9. Anticenters of the cyclic complex

Proposition 7. *Referring to Figure 9, the anticenters of the flanks build equal parallelograms with parallel sides:* $[lt][t][q][l]$, $[tlb][tqb][b][lb]$, $[q*][trb][rb][lbr]$ *and* $[ltr][rt][r][lqr]$. *The parallelogramic pattern is symmetric with respect to the middle* M *of segment* $[q][q*]$ *and the anticenters* $[tqb], [q], [q^*], [lqr]$ *are collinear. Besides the angles of the parallelograms are the same with the corresponding of the parallelogramic pattern of the centers.*

The proof is similar to the one of Proposition 2. For example, segments $[lt][t]$, $[l][q]$, $[tlb][tqb]$, $[lb][b]$, $[ltr][rt]$, $[lqr][r]$, $[q*][trb]$, $[lbr][rb]$ are all parallel since they are orthogonal to BC or its parallel B^*C^*.

A similar argument shows that the other sides are also parallel and also proves the statement about the angles. To prove the equality of parallelograms one can

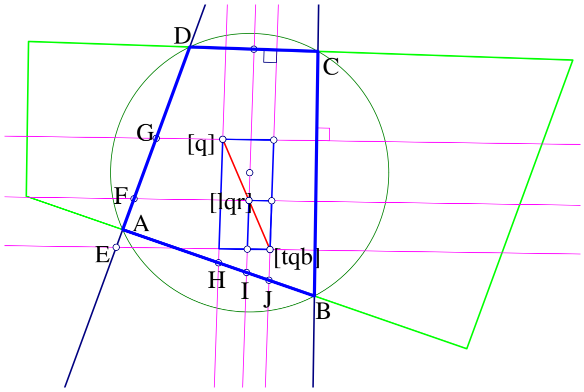

Figure 10. Collinearity of $[tqb]$, $[q]$, $[q^*]$, $[lqr]$

use again implied parallelograms, as, for example, $[lt][t][tqb][tlb]$, which shows the equality of horizontal sides of the two left parallelograms. The details can be completed as in Proposition 2. The only point where another kind of argument is needed is the collinearity assertion. For this, in view of the parallelities proven so far, it suffices to show that points $[q]$, $[lqr]$, $[tqb]$ are collinear. Figure 10 shows how this can be done. G, F, E, H, I, J are middles of sides of flanks, related to the definition of the three anticenters under consideration. It suffices to calculate the ratios and show that $|EF|/|EG| = |JI|/|JH|$. I omit the calculations.

5. Miscelanea

Here I will mention only a few consequences of the previous considerations and some supplementary properties of the complex, giving short hints for their proofs or simply figures that serve as hints.

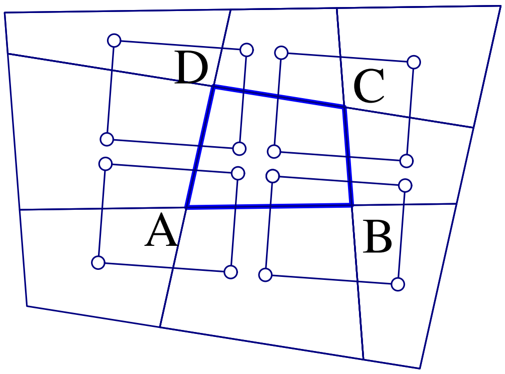

Figure 11. Barycenters of the flanks

Proposition 8. *The barycenters of the flanks build the pattern of equal parallelograms of Figure 11.*

Indeed, this is a consequence of the corresponding results for centers and anticenters of the flanks and the fact that *linear combinations* of parallelograms $c_i = (1-t)a_i + tb_i$, where a_i, b_j denote the vertices of parallelograms, are again parallelograms. Here $t = 1/2$, since the corresponding barycenter is the middle between center and anticenter. The equality of the parallelograms follows from the equality of corresponding parallelograms of centers and anticenters.

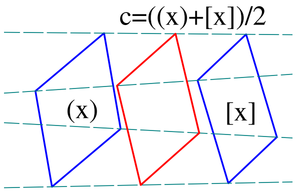

Figure 12. Linear combinations of parallelograms

Proposition 9. *Referring to Figure 13, the centers of the sides of the main flanks are aligned as shown and the corresponding lines intersect at the outer diagonal of q i.e. the line joining the intersection points of opposite sides of q.*

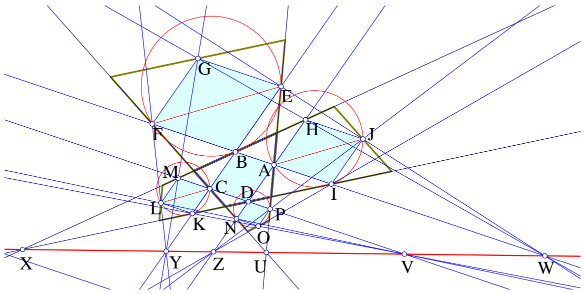

Figure 13. Lines of middles of main flanks

This is due to the fact that the main flanks are antihomothetic with respect to the vertices of q. Thus, the parallelograms of the main flanks are homothetic to each other and their homothety centers are aligned by three on a line. Later assertion can be reduced to the well known one for similarity centers of three circles, by considering the circumcircles of appropriate triangles, formed by parallel diagonals of the four parallelograms. The alignement of the four middles along the sides of $ABCD$ is due to the equality of angles of cyclic quadrilaterals shown in Figure 14.

Proposition 10. *Referring to Figure 15, the quadrilateral $(t)(r)(b)(l)$ of the centers of the main flanks is symmetric to the quadrilateral of the centers of the peripheral flanks $(tlb)(ltr)(trb)(lbr)$. The symmetry center is the middle of the line of $(q)(q^*)$.*

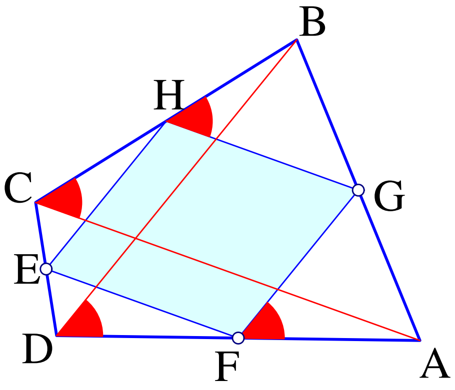

Figure 14. Equal angles in cyclic quadrilaterals

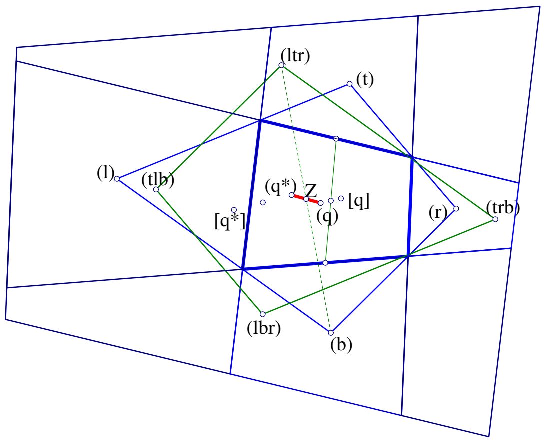

Figure 15. Symmetric quadrilaterals of centers

There is also the corresponding sort of dual for the anticenters, resulting by replacing the symbols (x) with $[x]$:

Proposition 11. *Referring to Figure 16, the quadrilateral* $[t][r][b][l]$ *of the anticenters of the main flanks is symmetric to the quadrilateral of the anticenters of the peripheral flanks* $[tlb][ltr][trb][lbr]$*. The symmetry center is the middle of the line of* $[q][q^*]$.

By the way, the symmetry of center and anticenter about the barycenter leads to a simple proof of the chararacteristic property of the anticenter. Indeed, consider the symmetric $A^*B^*C^*D^*$ of q with respect to the barycenter of q. The orthogonal from the middle of one side of q to the opposite one, to AD say, is also orthogonal to its symmetric A^*D^*, which is parallel to AD (Figure 17). Since A^*D^* is a chord

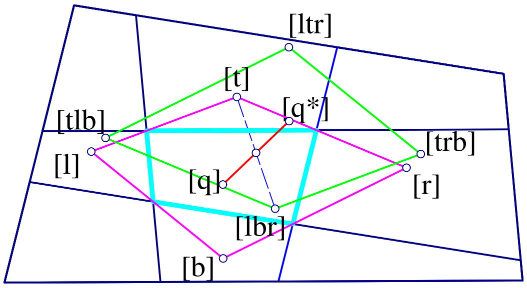

Figure 16. Symmetric quadrilaterals of anticenters

of the symmetric of the circumcircle, the orthogonal to its middle passes through the corresponding circumcenter, which is the anticenter.

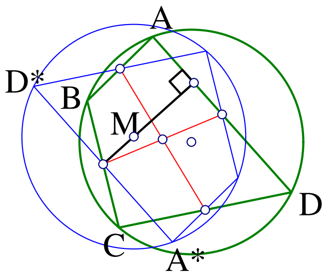

Figure 17. Anticenter's characteristic property

The following two propositions concern the radical axes of two particular pairs of circles of the complex:

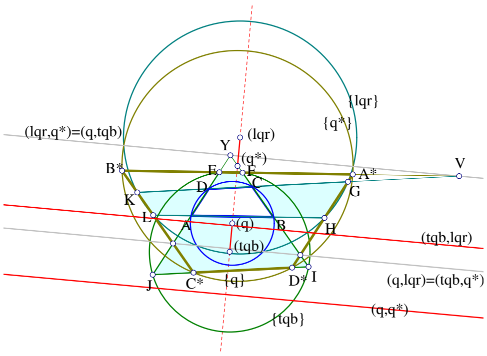

Figure 18. Harmonic bundle of radical axes

Proposition 12. *Referring to Figure 18, the radical axes* $(lqr, q^*) = (q, tqb)$ *and* $(q, lqr) = (tqb, q^*)$. *Besides the radical axes* (tqb, lqr) *and* (q, q^*) *are parallel to the previous two and define with them a harmonic bundle of parallel lines.*

Proposition 13. *Referring to Figure 19, the common tangent* (t, r) *is parallel to the radical axis* (tlb, lbr). *Analogous statements hold for the common tangents of the other pairs of adjacent main flanks.*

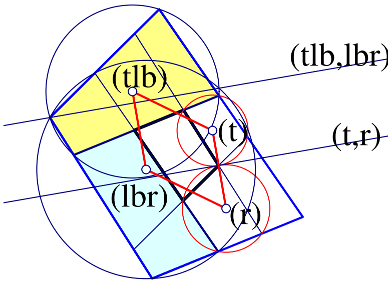

Figure 19. Common tangents of main flanks

6. Generalized complexes

There is a figure, similar to the cyclic complex, resulting in another context. Namely, when considering two arbitrary circles a, b and two other circles c, d tangent to the first two. This is shown in Figure 20. The figure generates a complex of quadrilaterals which I call a *generalized complex* of the cyclic quadrilateral. There are many similarities to the cyclic complex and one substantial difference, which prepares us for the discussion in the next paragraph. The similarities are:

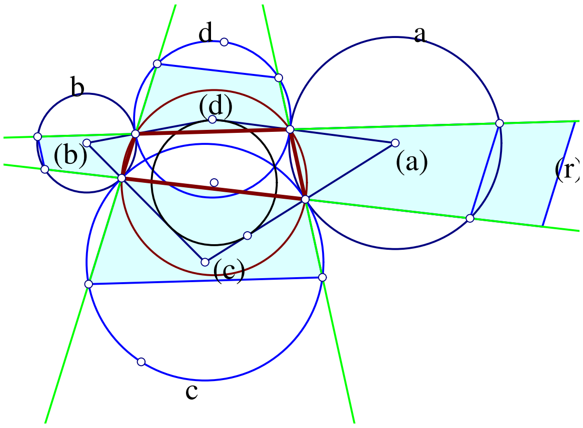

Figure 20. A complex similar to the cyclic one

(1) The points of tangency of the four circles define a cyclic quadrilateral q.
(2) The centers of the circles form a circumscriptible quadrilateral with center at the circumcenter of q.
(3) There are defined flanks, created by the other intersection points of the sides of q with the circles.
(4) Adjacent flanks are antihomothetic with homothety centers at the vertices of q.
(5) The same parallelogramic patterns appear for circumcenters, anticenters and barycenters.

Figure 21 depicts the parallelogrammic pattern for the circumcenters (in red) and the anticenters (in blue). Thus, the properties of the complex, discussed so far, could have been proved in this more general setting. The only difference is that the central cyclic quadrilateral q is not similar, in general, to the flanks, created in this way. In Figure 21, for example, the right cyclic-complex-flank r of q has been also constructed and it is different from the flank created by the general procedure.

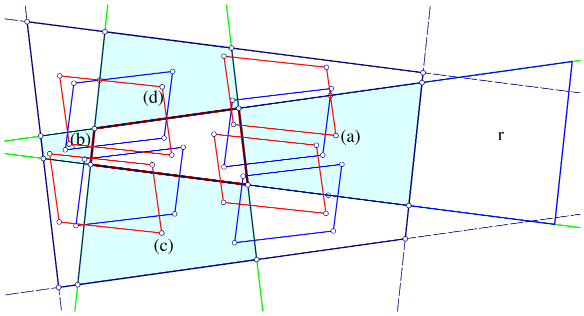

Figure 21. Circumcenters and anticenters of the general complex

Having a cyclic quadrilateral q, one could use the above remarks to construct infinite many generalized complexes (Figure 22) having q as their *central quadrilateral*. In fact, start with a point, F say, on the medial line of side AB of $q = ABCD$. Join it to B, extend FB and define its intersection point G with the medial of BC. Join G with C extend and define the intersection point H with the medial of CD. Finally, join H with D extend it and define I on the medial line of side DA. q being cyclic, implies that there are four circles centered, correspondingly, at points F, G, H and I, tangent at the vertices of q, hence defining the configuration of the previous remark.

From our discussion so far, it is clear, that the *cyclic complex* is a well defined complex, uniquely distinguished between the various generalized complexes, by the property of having its flanks similar to the original quadrilateral q.

7. The inverse problem

The inverse problem asks for the determination of q, departing from the big flank q^*. The answer is in the affirmative but, in general, it is not possible to construct q by elementary means. The following lemma deals with a completion of the figure handled in lemma-1.

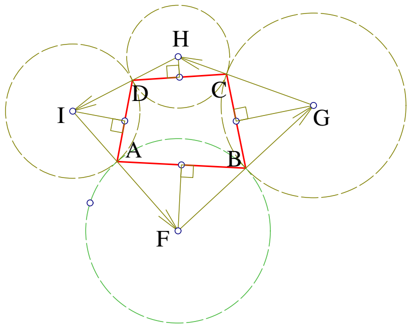

Figure 22. The generalized setting

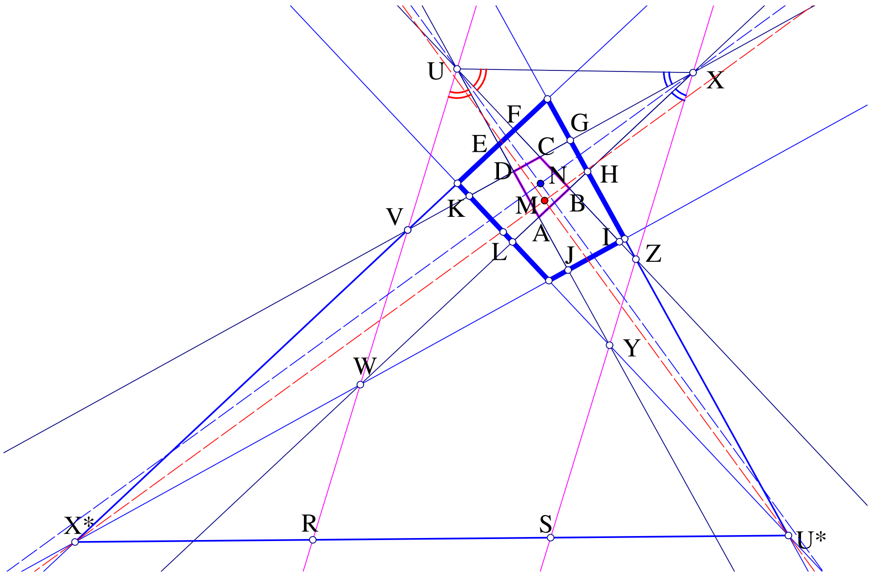

Figure 23. Lines XZY and UVW

Lemma 14. *Referring to Figure 23, lines XZY and UVW are defined by the intersection points of the opposite sides of main flanks of q. Line X^*U^* is defined by the intersection points of the opposite sides of q^*. The figure has the properties:*
(1) *Lines XZY and UVW are parallel and intersect line X^*U^* at points S, R trisecting segment X^*U^*.*
(2) *Triangles YKX, ZCX, UCV, UAX, UIV are similar.*
(3) *Angles $\widehat{VUD} = \widehat{CUX}$ and $\widehat{ZXH} = \widehat{GXU}$.*
(4) *The bisectors of angles $\widehat{VUX}, \widehat{UXZ}$ are respectively identical with those of $\widehat{DUC}, \widehat{DXA}$.*

(5) *The bisectors of the previous angles intersect orthogonaly and are parallel to the bisectors X^*M, U^*M of angles $\widehat{VX^*W}$ and $\widehat{YU^*Z}$.*

(1) is obvious, since lines UVW and XZY are diagonals of the parallelograms X^*WXV and U^*YUZ. (2) is also trivial since these triangles result from the extension of sides of similar quadrilaterals, namely q and its main flanks. (3) and (4) is a consequence of (2). The orthogonality of (5) is a general property of cyclic quadrilaterals and the parallelity is due to the fact that the angles mentioned are opposite in parallelograms.

The lemma suggests a solution of the inverse problem: Draw from points R, S two parallel lines, so that the parallelograms X^*WXV and U^*YUZ, with their sides intersections, create $ABCD$ with the required properties. Next proposition investigates a similar configuration for a general, not necessarily cyclic, quadrangle.

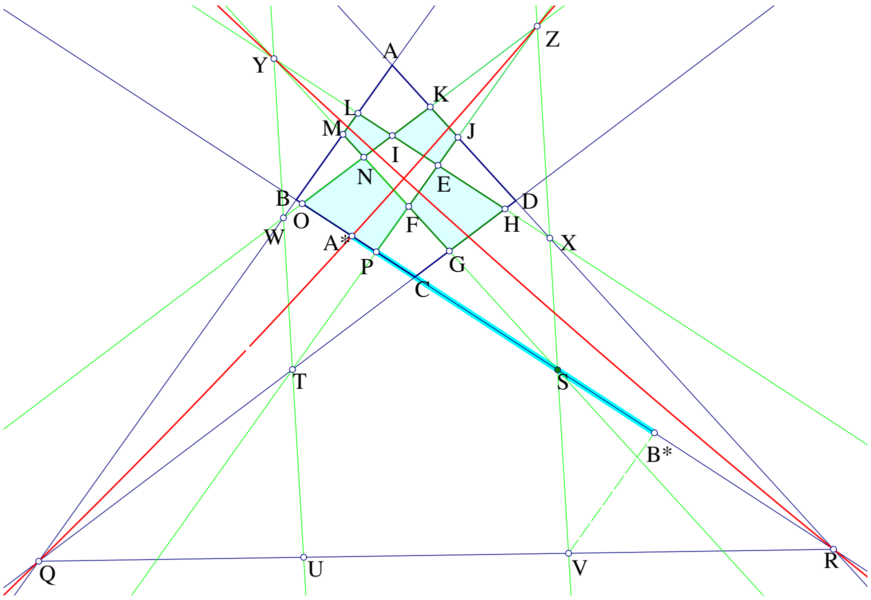

Figure 24. Inverse problem

Proposition 15. *Referring to Figure 24, consider a quadrilateral $q^* = ABCD$ and trisect the segmet QR, with end-points the intersections of opposite sides of q^*. From trisecting points U, V draw two arbitrary parallels UY, VX intersecting the sides of q^* at W, T and S, X respectively. Define the parallelograms $QWZT, RSYX$ and through their intersections and the intersections with q^* define the central quadrilateral $q = EFNI$ and its flanks $FEHG, FPON, NMLI, IKJE$ as shown.*

(1) *The central quadrilateral q has angles equal to q^*. The angles of the flanks are complementary to those of q^*.*

(2) *The flanks are always similar to each other, two adjacent being anti-homothetic*

with respect to their common vertex. (3) *There is a particular direction of the parallels, for which the corresponding central quadrilateral q has side-lengths-ratio $|EF|/|FN| = |ON|/|OP|$.*

In fact, (1) is trivial and (2) follows from (1) and an easy calculation of the ratios of the sides of the flanks. To prove (3) consider point S varying on side BC of q^*. Define the two parallels and in particular VX by joining V to S. Thus, the two parallels and the whole configuration, defined through them, becomes dependent on the location of point S on BC. For S varying on BC, a simple calculation shows that points Y, Z vary on two hyperbolas (red), the hyperbola containing Z intersecting BC at point A^*. Draw VB^* parallel to AB, B^* being the intersection point with BC. As point S moves from A^* towards B^* on segment A^*B^*, point Z moves on the hyperbola from A^* to infinity and the cross ratio $r(S) = \frac{|EF|}{|FN|} : \frac{|ON|}{|OP|}$ varies increasing continuously from 0 to infinity. Thus, by continuity it passes through 1.

Proposition 16. *Given a circular quadrilateral $q^* = A^*B^*C^*D^*$ there is another circular quadrilateral $q = ABCD$, whose cyclic complex has corresponding big flank the given one.*

The proof follows immediately by applying (3) of the previous proposition to the given cyclic quadrilateral q^*. In that case, the condition of the equality of ratios implies that the constructed by the proposition central quadrilateral q is similar to the main flanks. Thus the given q^* is identical with the big flank of q as required.

Remarks. (1) Figure 24 and the related Proposition 14 deserve some comments. First, they show a way to produce a complex out of any quadrilateral, not necessary a cyclic one. In particular, condition (3) of the aforementioned proposition suggests a unified approach for general quadrilaterals that produces the cyclic complex, when applied to cyclic quadrilaterals. The suggested procedure can be carried out as follows (Figure 25): (a) Start from the given general quadrilateral $q = ABCD$ and construct the first flank $ABFE$ using the restriction $|AE|/|EF| = |BC|/|AB| = k_1$. (b) Use appropriate anti-homotheties centered at the vertices of q to transplant the flank to the other sides of q. These are defined inductively. More precisely, having flank-1, use the anti-homothety $(B, \frac{|BC|}{|FB|})$ to construct flank-2 $BGHC$. Then repeat with analogous constructions for the two remaining flanks. It is easy to see that this procedure, applied to a cyclic quadrilateral, produces its cyclic complex, and this independently from the pair of adjacent sides of q, defining the ratio k_1. For general quadrilaterals though the complex depends on the initial choice of sides defining k_1. Thus defining $k_1 = |DC|/|BC|$ and starting with flank-2, constructed through the condition $|BG|/|GH| = k_1$ etc. we land, in general, to another complex, different from the previous one. In other words, the procedure has an element of arbitrariness, producing four complexes in general, depending on which pair of adjacent sides of q we start it.

(2) The second remark is about the results of Proposition 8, on the centroids or barycenters of the various flanks. They remain valid for the complexes defined

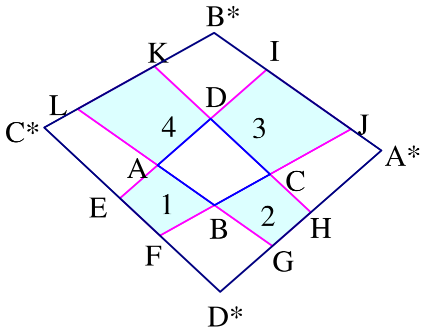

Figure 25. Flanks in arbitrary quadrilaterals

through the previously described procedure. The proof though has to be modified and given more generally, since circumcenters and anticenters are not available in the general case. The figure below shows the barycenters for a general complex, constructed with the procedure described in (1).

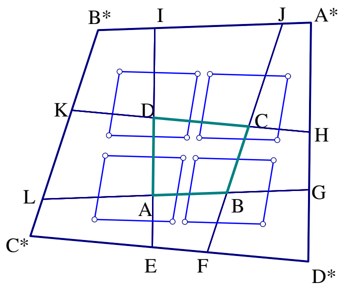

Figure 26. Barycenters for general complexes

An easy approach is to use vectors. Proposition-9, with some minor changes, can also be carried over to the general case. I leave the details as an exercise.
(3) Although Proposition 15 gives an answer to the existence of a sort of *soul* (q) of a given cyclic quadrilateral (q^*), a more elementary constuction of it is desirable. Proposition-14, in combination with the first remark, shows that even general quadrilaterals have *souls*.
(4) One is tempted to look after the soul of a soul, or, stepping inversely, the complex and corresponding big flank of the big flank etc.. Several questions arise in this context, such as (a) are there repetitions or periodicity, producing something similar to the original after a finite number of repetitions? (b) which are the limit points, for the sequence of souls?

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