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Archimedean Circles Related to the Schoch Line

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Abstract. We give several Archimedean circles of the arbelos related to the Schoch line.

Let us consider an arbelos consisting of three semicircles α, β and γ with diameters AO, BO and AB, respectively, where O is a point on the segment AB. Thomas Schoch has considered the circles α' and β' with centers A and B and passing through the point O. He has found that the circle touching the two circles externally and the semicircle γ internally is Archimedean [1]. The perpendicular to AB from the center of this circle is called the Schoch line (see Figure 1). In this note we give several Archimedean circles touching this line or one of the two circles.

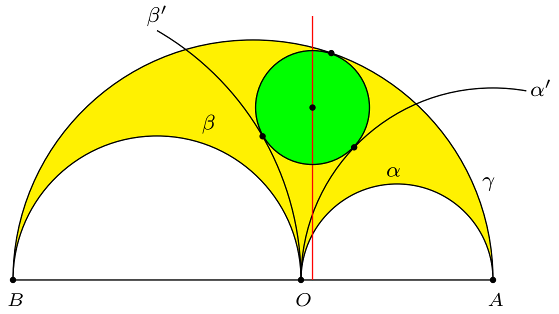

Figure 1

Let a and b be the radii of α and β, respectively. The radius of Archimedean circles is $\frac{ab}{a+b}$, which is denoted by r_A. We use a rectangular coordinate system with origin O such that the points A and B have coordinates $(2a, 0)$ and $(-2b, 0)$, respectively, where we assume that all the semicircles are constructed in the region $y > 0$. Let $s = r_A \cdot \frac{b-a}{b+a}$. The Schoch line is expressed by the equation $x = s$ (see [2]). If $b > a$, then $s > 0$, and the Schoch line intersects the semicircle α.

Let O_α and O_β be the centers of α and β respectively, and μ the circle with $O_\alpha O_\beta$ as diameter (see Figure 2).

Theorem 1. (1) *The two circles each touching the circle μ and α* (respectively β)*, all externally, and the Schoch line from the side opposite to B* (respectively A) *are Archimedean.*

(2) *If $b > a$, then the circle touching the circle μ internally, α externally, and the Schoch line from the side opposite to A is Archimedean.*

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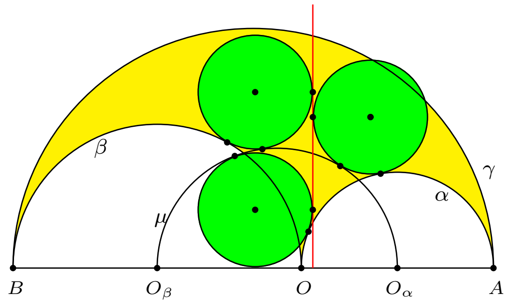

Figure 2

Proof. Let x be the radius of the circle touching the circle μ and α externally, and the Schoch line from side opposite to B in (1). By the Pythagorean theorem,

$$(a+x)^2-(s+x-a)^2=\left(\frac{a+b}{2}+x\right)^2-\left(s+x-\frac{a-b}{2}\right)^2.$$

Solving the equation for x, we get $x=r_A$, i.e., the circle is Archimedean. The rest of the theorem is proved similarly. $\square$

Let $\mathcal{L}_\alpha$ be the perpendicular to AB from the point of intersection of γ and α'. The line $\mathcal{L}_\beta$ is defined similarly. Each of the two lines touches one of the twin circles of Archimedes [1] (see Figure 3). The proof of the following theorem is similar to that of Theorem 1, and is omitted.

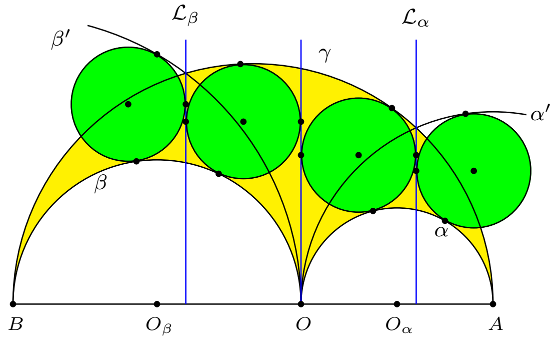

Figure 3

Theorem 2. *Each of the circles touching α externally, α' internally, and $\mathcal{L}_\alpha$ from the side opposite to O* (respectively β, β' and $\mathcal{L}_\beta$) *is Archimedean.*

References

[1] C. W. Dodge, T. Schoch, P. Y. Woo, and P. Yiu, Those ubiquitous Archimedean circles, *Math. Mag.*, 72 (1999) 202–213.

[2] H. Okumura and M. Watanabe, The Archimedean circles of Schoch and Woo, *Forum Geom.*, 4 (2004) 27–34.

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