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Another synthetic proof of the butterfly theorem using the midline in triangle

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Abstract. We give a new synthetic proof of the butterfly theorem, based on the use of midline in triangle, and cyclic quadrilateral.

This article is to give a new proof of the butterfly theorem.

Butterfly Theorem. *Let M be the midpoint of a chord AB of a circle. Through M two other chords CD and EF are drawn. If C and F are on opposite sides of AB, and CF, DE intersect AB at G and H respectively, then M is also the midpoint of GH.*

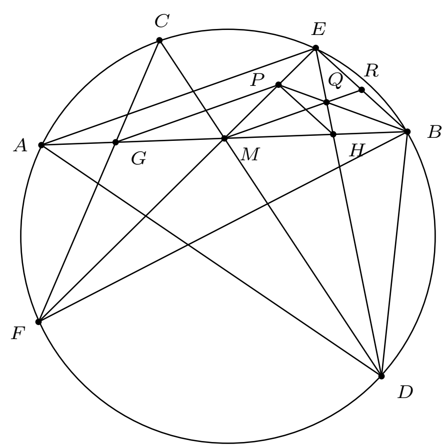

Figure 1

Let P be the point on segment ME such that $GP \parallel AE$. PB intersects EH at Q. We have $\angle PGB = \angle EAB = \angle EFB = \angle PFB$. This shows that quadrilateral $FGPB$ is cyclic. We get $\angle QBM = \angle PBG = \angle PFG = \angle EFC = \angle EDC = \angle QDM$. Therefore, quadrilateral $DMQB$ is also cyclic. From this, $\angle QMB = \angle QDB = \angle EDB = \angle EAB$, and $MQ \parallel AE$. Since M is the midpoint of AB, by the midline theorem, MQ passes through the midpoint R of EB. By Ceva's theorem for triangle MEB and Thales's theorem for triangle MEA, we get $\frac{MH}{MB} = \frac{MP}{ME} = \frac{MG}{MA}$. Since $MA = MB$, we have $MG = MH$.

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