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Another Simple Construction of the Golden Section with Equilateral Triangles

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Abstract. A simple construction of the golden section with equilateral triangles.

Three congruent equilateral triangles $A_1B_1C_1$, $A_2B_2C_2$, and $A_3B_3C_3$ are positioned in such a way that B_1 and B_3 are respectively midpoints of the segments A_2C_2 and A_2B_2, while the vertices A_1, A_2, A_3 are on a line perpendicular to both the segments A_1C_1 and A_2C_2.

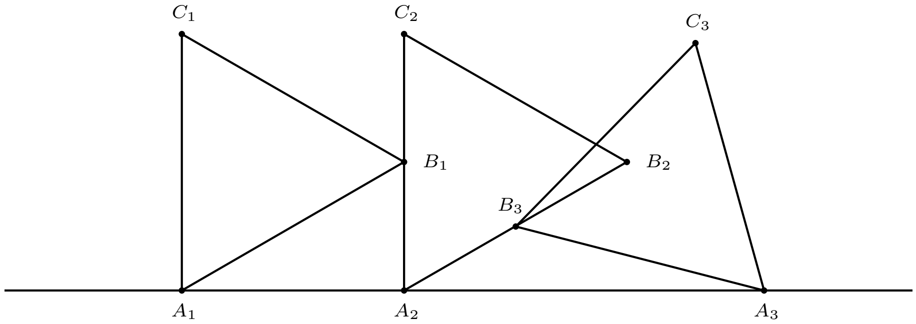

Figure 1

In this arrangement, A_2 divides A_1A_3 in the golden ratio.

Proof. Let a be a side of the equilateral triangles. Because $\angle B_1A_1A_2 = 30°$,

$$A_1A_2 = a\cos 30° = \frac{\sqrt{3}}{2}a. \tag{1}$$

Again, since $\angle B_3A_2A_3 = 30°$, by the law of cosines,

$$\begin{aligned} A_3B_3^2 &= A_2B_3^2 + A_2A_3^2 - 2A_2B_3 \cdot A_2A_3 \cos 30°, \\ a^2 &= \frac{a^2}{4} + A_2A_3^2 - 2 \cdot \frac{a}{2} \cdot A_2A_3 \cdot \cos 30° \\ \frac{3}{4}a^2 &= A_2A_3^2 - A_2A_3 \cdot a\cos 30°. \end{aligned}$$

From (1), $A_1A_2^2 = A_2A_3^2 - A_1A_2 \cdot A_2A_3$. Hence,

$$\frac{A_2A_3}{A_1A_2} = \frac{\sqrt{5}+1}{2}.$$

□

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Reference.

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