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Irrationality of $\sqrt{2}$: Yet Another Visual Proof

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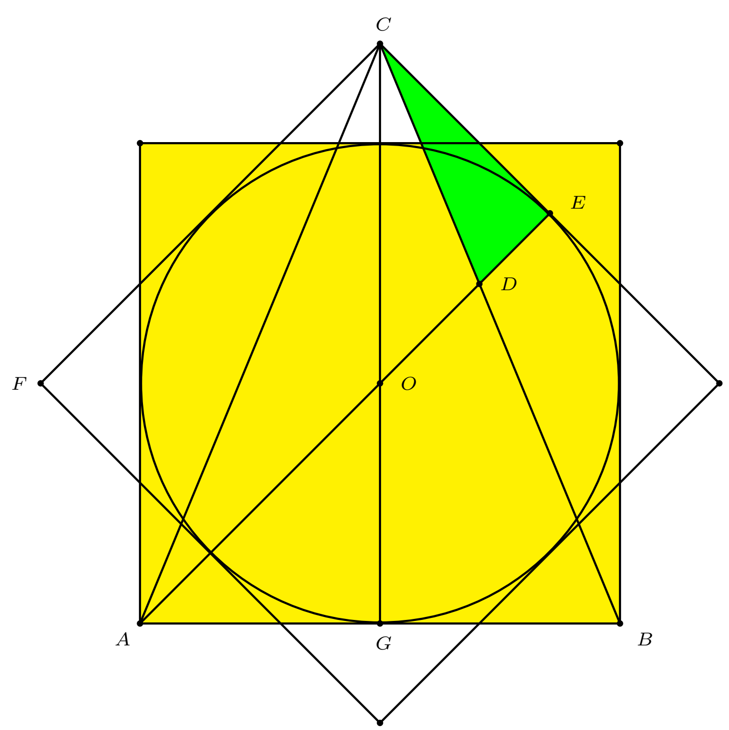
Abstract. Another visual proof of the irrationality of $\sqrt{2}$.

Two identical right triangles intersect as shown in the figure resulting in a smaller right triangle which happens to be, as is easy to check, similar to the original ones.



Since $\overline{AE}$ and $\overline{CF}$ are parallel line segments, $\angle EAC = \angle ACF$. By symmetry, $\angle ACF = \angle DCE$ so that triangles DCE and CAE are similar. Moreover, $\angle ADB = \angle CDE$, but by similarity $\angle CDE = \angle ACE$. Since $\angle ACE = \angle CBG = \angle DBA$, it follows that triangle ADB is isosceles.

The ratio of the lengths of the legs in the right triangles ACE and CDE is $(\overline{AO} + \overline{OE})/\overline{CE} = (\overline{OC} + \overline{OE})/\overline{OE} = \sqrt{2} + 1$. If $\sqrt{2}$ is rational, so is $\sqrt{2} + 1$, and thus $\overline{AE} = m$ and $\overline{CE} = n$ for some positive integers m, n. Since ABD is

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isosceles, then $\overline{DE} = \overline{AE} - \overline{AD} = \overline{AE} - \overline{AB} = \overline{AE} - 2\overline{CE} = m - 2n$. This process may be repeated indefinitely, triggering an infinite decreasing sequence of positive integers $m > n > m - 2n > 5n - 2m > \cdots$. But this is impossible. Thus, $\sqrt{2}$ cannot be rational.

Reference

[1] A. Bogomolny, Square root of two is irrational,
`http://www.cut-the-knot.org/ proofs/sq_root.shtml`.

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