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A Remark on the Arbelos and the Regular Star Polygon

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Abstract. We give a condition that a regular star polygon $\{\frac{n}{2}\}$ can be constructed from an arbelos.

We consider to construct a regular star polygon $\{\frac{n}{2}\}$ from an arbelos. Let us consider an arbelos made by the three circles α, β, γ with diameters AO, BO, AB, respectively, for a point O on the segment AB. Circles of radius $ab/(a+b)$ are called Archimedean circles, where a and b are the radii of α and β, respectively. We call the perpendicular to AB at O the axis.

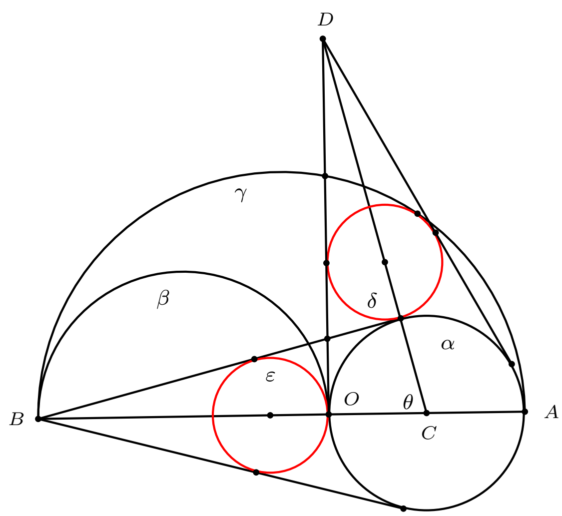

Figure 1.

The circle touching γ internally, α externally and the axis is Archimedean [1], which we describe by δ (see Figure 1). While the circle touching β internally and the tangents of α from B is also Archimedean [1], which is denoted by ε. Therefore the figure made by δ, α and their tangents are congruent to the figure made by ε, α and their tangents. We assume that C is the center of α, D is the external center of similitude of α and δ, and $\theta = \angle BCD$. The congruence of the two figures implies that we can construct a regular star polygon $\{\frac{n}{2}\}$ with center C and adjacent vertices B and D if $\theta = 2\pi/n$, while $\cos\theta = a/(a+2b)$. Therefore

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we can construct a regular star polygon $\{\frac{n}{2}\}$ *with center* C *and adjacent vertices* B *and* D *if and only if*

$$\frac{a}{a+2b} = \cos\frac{2\pi}{n}.$$

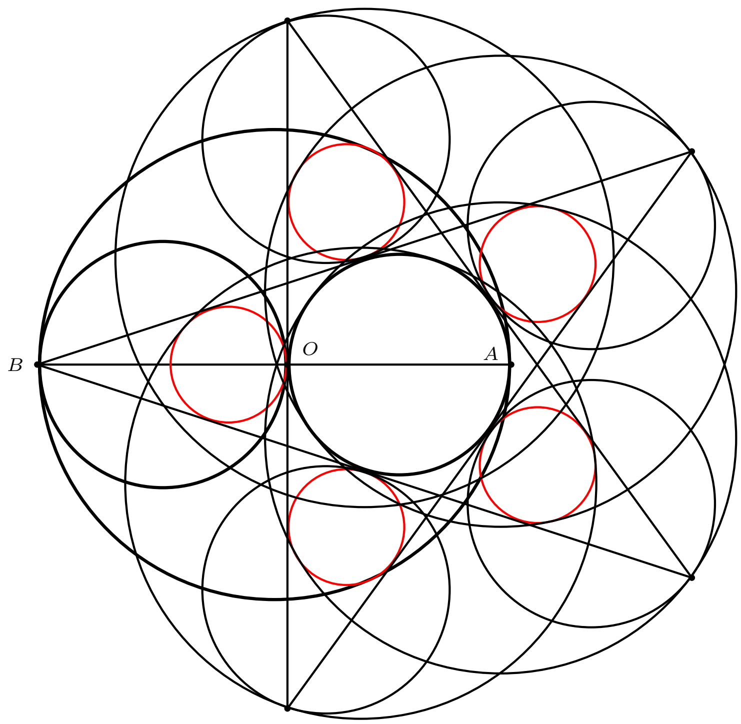

Figure 2. $\left\{\frac{5}{2}\right\}$, $\quad b = \sqrt{5}a/2$

Figure 2 shows the case $n = 5$. The distance between D and the point of contact of α and δ equals BO by the congruence. A problem stating this fact can be found in [2].

References

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[2] T. Hermann, Twin segments in the arbelos, Solution to Problem 10895, *Amer. Math. Monthly*, 110 (2003) 63–64.

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