**Exercice 1** Six points are chosen on the sides of an equilateral triangle $ABC$ : $A_1, A_2$ on $BC$, $B_1, B_2$ on $CA$ and $C_1, C_2$ on $AB$, such that they are the vertices of a convex hexagon $A_1A_2B_1B_2C_1C_2$ with equal side lengths. Prove that the lines $A_1B_2$, $B_1C_2$ et $C_1A_2$ are concurrent.

**Exercice 2** Let $a_1, a_2, \ldots$ be a sequence of integers with infinitely many positive and negative terms. Suppose that for every positive integer $n$, the numbers $a_1, a_2, \ldots, a_n$ leave $n$ different remainders upon division by $n$. Prove that every integer occurs exactly once in the sequence $a_1, a_2, \ldots$

**Exercice 3** Let $x, y, z$ be three positive reals such that $xyz \geq 1$. Prove that

$$\frac{x^5 - x^2}{x^5 + y^2 + z^2} + \frac{y^5 - y^2}{y^5 + z^2 + x^2} + \frac{z^5 - z^2}{z^5 + x^2 + y^2} \geq 0.$$

**Exercice 4** Determine all positive integers relatively prime to all the terms of the infinite sequence $a_n = 2^n + 3^n + 6^n - 1$, $n \geq 1$.

**Exercice 5** Let $ABCD$ be a fixed convex quadrilateral with $BC = DA$ and $BC$ not parallel with $DA$. Let two variable points $E$ and $F$ lie on the sides $BC$ and $DA$ respectively and satisfy $BE = DF$. The lines $AC$ and $BD$ meet at $P$, the lines $BD$ and $EF$ meet at $Q$, the lines $EF$ and $AC$ meet at $R$. Prove that the circumcircles of the triangles $PQR$, as $E$ and $F$ vary, have a common point other than $P$.

**Exercice 6** In a mathematical competition in which 6 problems were posed to the participants, every two of these problems were solved by more than $\frac{2}{5}$ of the contestants. Moreover, no contestant solved all the 6 problems. Show that there is at least 2 contestants who solved exactly 5 problems each.